

Development and Validation of Crack Growth Models and Life Enhancement Methods for Rotorcraft Damage Tolerance

Center for Aerospace Research & Education
University of California Irvine

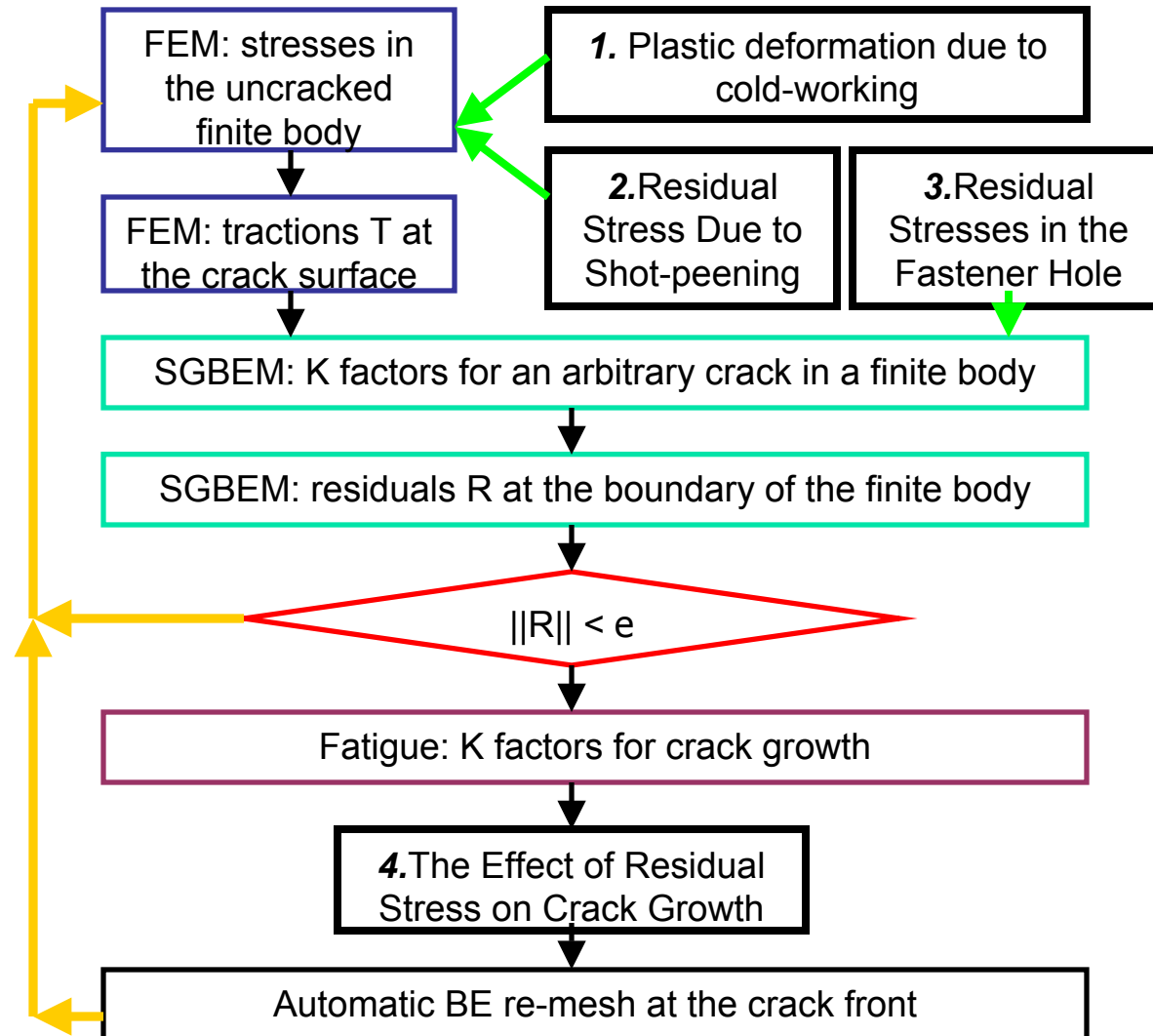
Objectives

The objective of this research is to **develop**, **validate**, and **demonstrate** crack growth analysis models and life enhancement .

The life enhancement methods that are being considered in this project are:

1. **Rivet mis-fitting**;
2. **cold working of rivet-holes**;
3. **shot peening**.

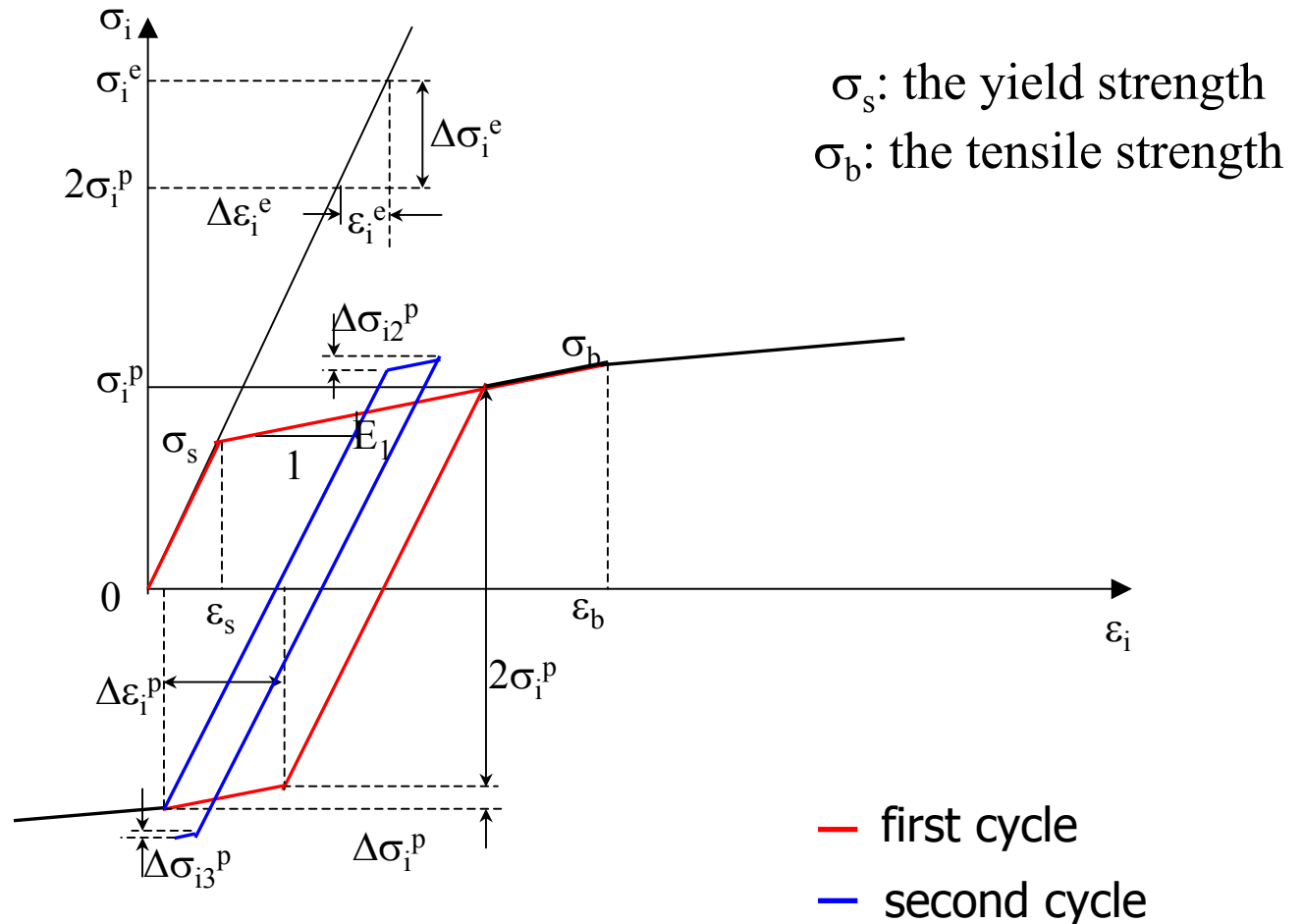
Schematic of Analysis with effects of residual stress fields



Analytical model to model the shot-peening process with 200% coverage

- ▶ Elastic analysis of the loading process; Hertzian contact theory-----Determine a_e , and the total elastic field.
- ▶ Elastic-plastic analysis of the loading process; Multilinear stress-strain relationship-----Determine a_p .
- ▶ Thus, a_e & a_p are determined for a given V & R of the shot.
- ▶ We assume that the ratio of ε_i^p to ε_i^e on the z-axis inside the target is equal to the ratio α , $\alpha = a_e / a_p$, of the deformation at the surface.

Schematic diagram for calculating residual stress-200% Coverage



Isotropic hardening

Elastic-plastic analysis

The strain deviations

$$e_x^p = e_y^p = \frac{1}{3}(1+\nu)\varepsilon_i^p$$

$$e_z^p = -\frac{2}{3}(1+\nu)\varepsilon_i^p = -2e_x^p$$

The stress deviations

$$s_x^p = s_y^p = \frac{1}{1+\nu} \frac{\sigma_i^p}{\varepsilon_i^p} e_x^p = \frac{1}{3} \sigma_i^p$$

$$s_z^p = -\frac{2}{3} \sigma_i^p = -2s_x^p$$

Loading Process —the first shot

Elastic-plastic equivalent strain

$$\varepsilon_i^p = \begin{cases} \varepsilon_i^e & \text{for } \varepsilon_i^e \leq \varepsilon_s \\ \varepsilon_s + \alpha(\varepsilon_i^e - \varepsilon_s) & \text{for } \varepsilon_i^e > \varepsilon_s \end{cases}$$

Elastic-plastic equivalent stress


$$\sigma_i^p = \begin{cases} \sigma_i^e & \text{for } \varepsilon_i^p < \varepsilon_s \\ \sigma_s + E_1(\varepsilon_i^p - \varepsilon_s) & \text{for } \varepsilon_s \leq \varepsilon_i^p < \varepsilon_b \\ \sigma_b & \text{for } \varepsilon_i^p \geq \varepsilon_b \end{cases}$$

Unloading process —the first shot

Elastic-plastic equivalent stress after unloading

$$\sigma_{i1}^p = \begin{cases} 0 & \text{for } \sigma_i^e \leq \sigma_s \\ \sigma_i^p - \sigma_i^e & \text{for } \sigma_s \leq \sigma_i^e < 2\sigma_i^p \\ \sigma_i^p - 2\sigma_i^p - \Delta\sigma_i^p & \text{for } \sigma_i^e > 2\sigma_i^p \end{cases}$$

$$\Delta\sigma_i^e = \sigma_i^e - 2\sigma_i^p \longrightarrow \Delta\varepsilon_i^e = \frac{\Delta\sigma_i^e}{E}$$



$$\Delta\varepsilon_i^p = \alpha\Delta\varepsilon_i^e \longrightarrow \Delta\sigma_i^p$$

Reloading process —the second shot

Elastic-plastic equivalent stress after reloading

$$\sigma_{i2}^p = \begin{cases} \sigma_{i1}^p + \sigma_i^e & \text{for } 2\sigma_i^p < \sigma_i^e \leq -2\sigma_{i1}^p \\ -\sigma_{i1}^p + \Delta\sigma_{i2}^p & \text{for } -2\sigma_{i1}^p \leq \sigma_i^e \end{cases}$$

$$\Delta\sigma_{i2}^e = \sigma_i^e + 2\sigma_{i1}^p \longrightarrow \Delta\varepsilon_{i2}^e = \frac{\Delta\sigma_{i2}^e}{E}$$



$$\Delta\varepsilon_{i2}^p = \alpha\Delta\varepsilon_{i2}^e \longrightarrow \Delta\sigma_{i2}^p$$

Unloading process —the second shot

Elastic-plastic equivalent stress after unloading

$$\sigma_{i3}^p = \begin{cases} \sigma_{i1}^p & \text{for } 2\sigma_i^p < \sigma_i^e \leq -2\sigma_{i1}^p \\ \sigma_{i2}^p - \sigma_i^e & \text{for } -2\sigma_{i1}^p \leq \sigma_i^e < 2\sigma_{i2}^p \\ \sigma_{i2}^p - 2\sigma_{i2}^p - \Delta\sigma_{i3}^p & \text{for } \sigma_i^e > 2\sigma_{i2}^p \end{cases}$$

$$\Delta\sigma_{i3}^e = \sigma_i^e - 2\sigma_{i2}^p \longrightarrow \Delta\varepsilon_{i3}^e = \frac{\Delta\sigma_{i3}^e}{E}$$



$$\Delta\varepsilon_{i3}^p = \alpha\Delta\varepsilon_{i3}^e \longrightarrow \Delta\sigma_{i3}^p$$

Residual stress after two shots

The residual stresses after two shots

$$\sigma_{ij}^r = \begin{cases} 0 & \text{for } \sigma_i^e \leq \sigma_s \\ s_{ij}^p - s_{ij}^e & \text{for } \sigma_s \leq \sigma_i^e < 2\sigma_i^p \end{cases}$$

$$\sigma_x^r = \sigma_y^r = \frac{1}{3}(\sigma_i^p - \sigma_i^e) \text{ for } \sigma_s \leq \sigma_i^e \leq 2\sigma_i^p, \quad \sigma_z^r = -2\sigma_x^r$$

$$\sigma_{ij}^r = \begin{cases} \frac{1}{3}\sigma_{i1}^p & \text{for } 2\sigma_i^p < \sigma_i^e \leq -2\sigma_{i1}^p \\ \frac{1}{3}(\sigma_{i2}^p - \sigma_i^e) & \text{for } -2\sigma_{i1}^p \leq \sigma_i^e < 2\sigma_{i2}^p \\ \frac{1}{3}(\sigma_{i2}^p - 2\sigma_{i2}^p - \Delta\sigma_{i3}^p) & \text{for } \sigma_i^e > 2\sigma_{i2}^p \end{cases}$$

Residual stress field for 200% coverage

The residual stress and strain fields should satisfy

$$\sigma_x^R = \sigma_y^R = f(z), \quad \sigma_z^R = 0$$

$$\varepsilon_x^R = \varepsilon_y^R = 0, \quad \varepsilon_z^R = g(z)$$

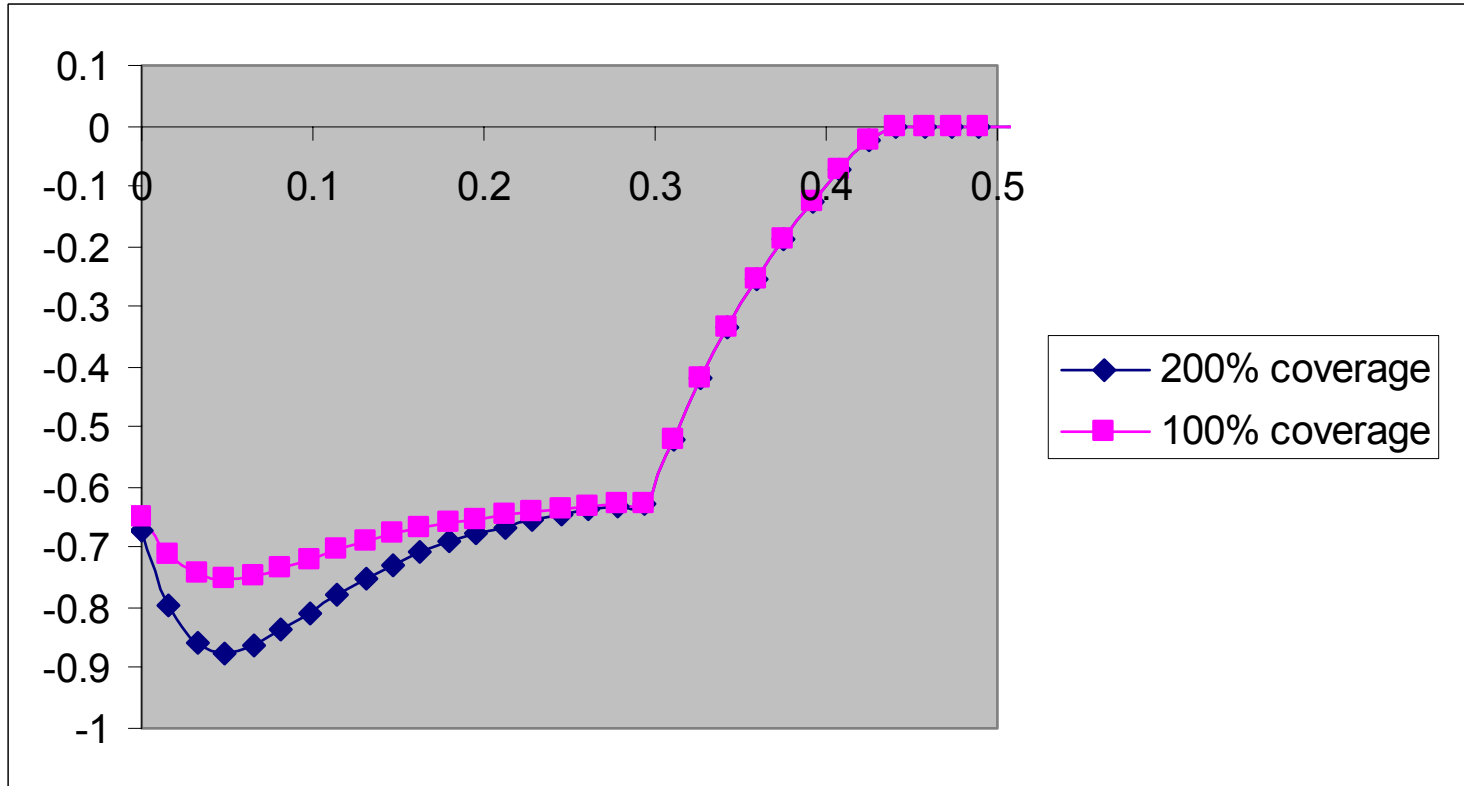
The relaxation values of σ_{ij}^r can be calculated by Hooke's law as

$$\sigma_x' = \sigma_y' = \frac{\nu}{1-\nu} \sigma_z^r$$

The final residual stress field is

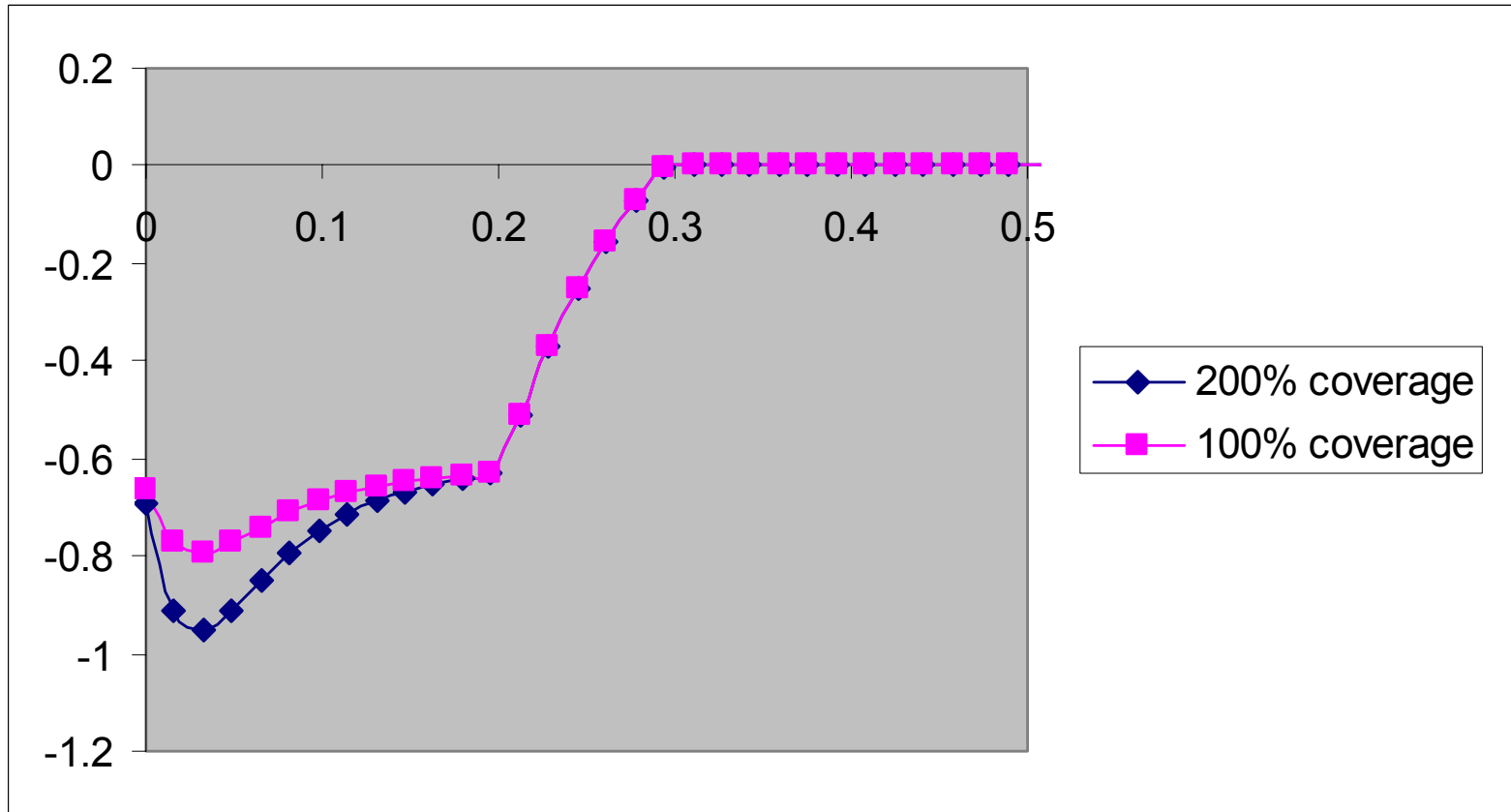
$$\sigma_x^R = \sigma_y^R = \sigma_x^r - \frac{\nu}{1-\nu} \sigma_z^r = \frac{1+\nu}{1-\nu} \sigma_x^r$$

The effect of the coverage



$R=0.55\text{mm}$, $E=200\text{GPa}$, $\nu=0.3$, $\rho=7800\text{kg/m}^3$, $V=30.65\text{m/s}$
 $\sigma_s=0.70\text{GPa}$, $\sigma_b=0.885\text{GPa}$, $\varepsilon_b=0.140$
40Cr Steel

The effect of the coverage

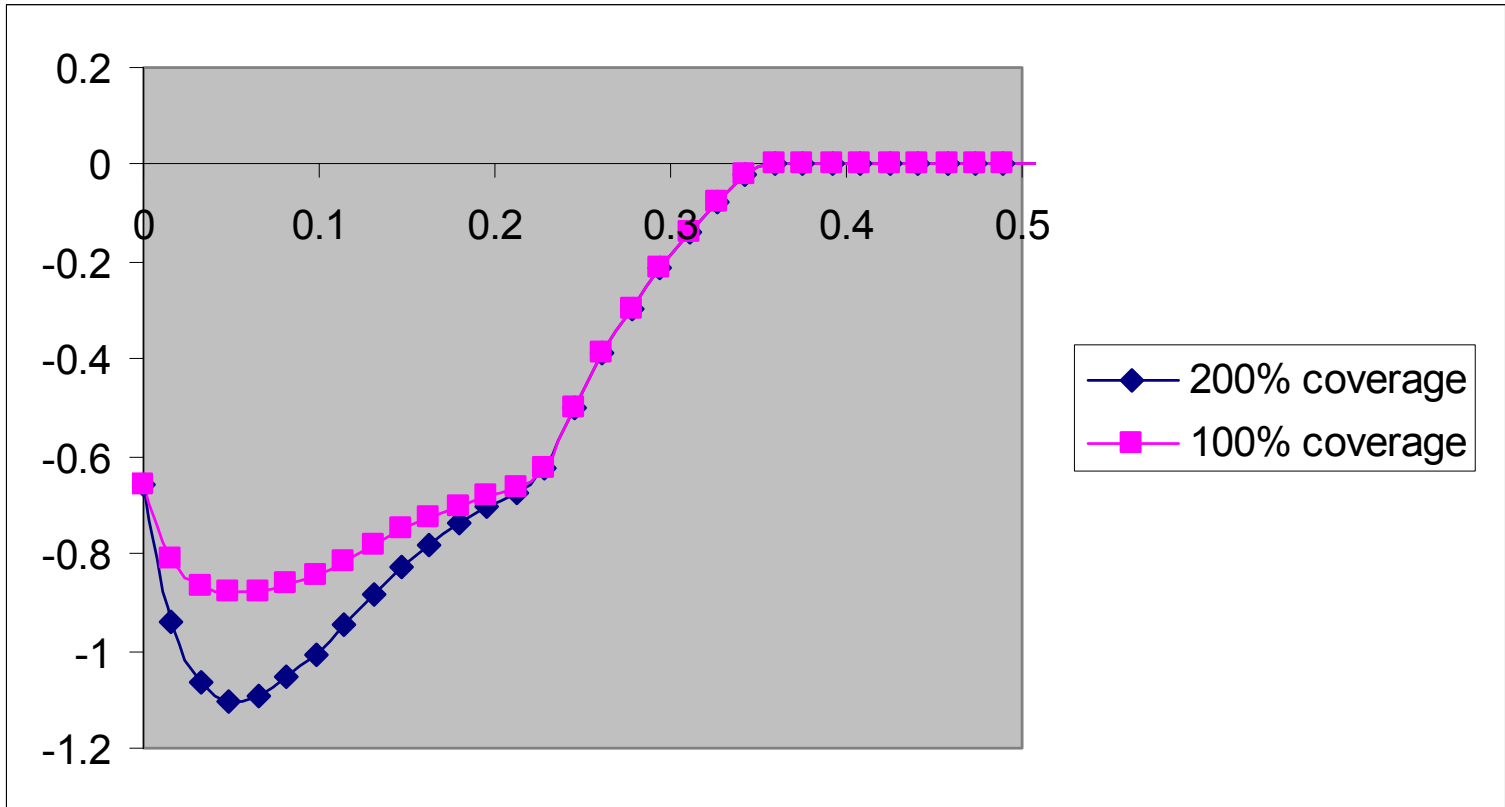


$R=0.275\text{mm}$, $E=200\text{GPa}$, $\nu=0.3$, $\rho=7800\text{kg/m}^3$, $V=50.44\text{m/s}$

$\sigma_s=0.70\text{GPa}$, $\sigma_b=0.885\text{GPa}$, $\varepsilon_b=0.140$

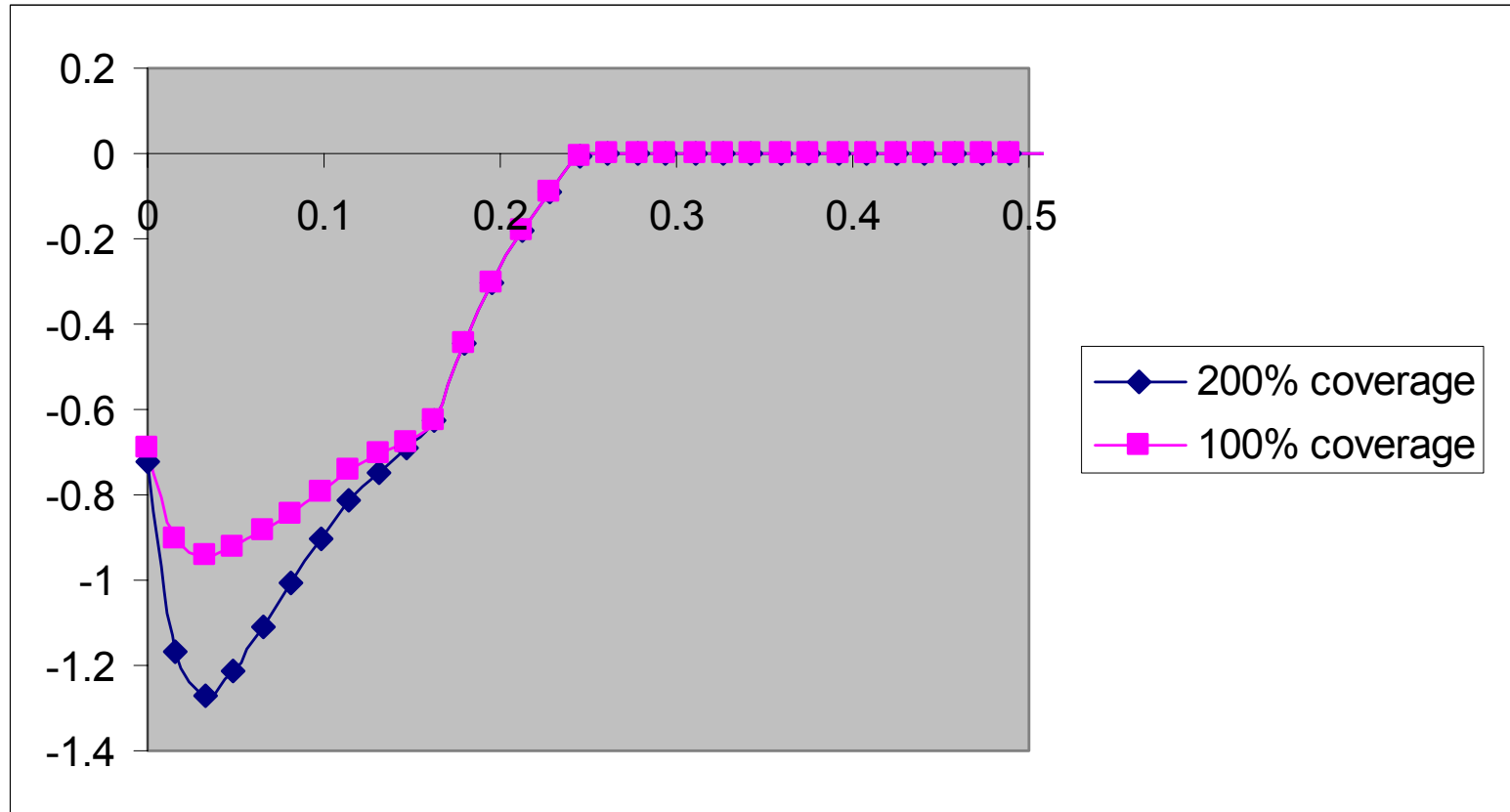
40Cr Steel

The effect of the coverage



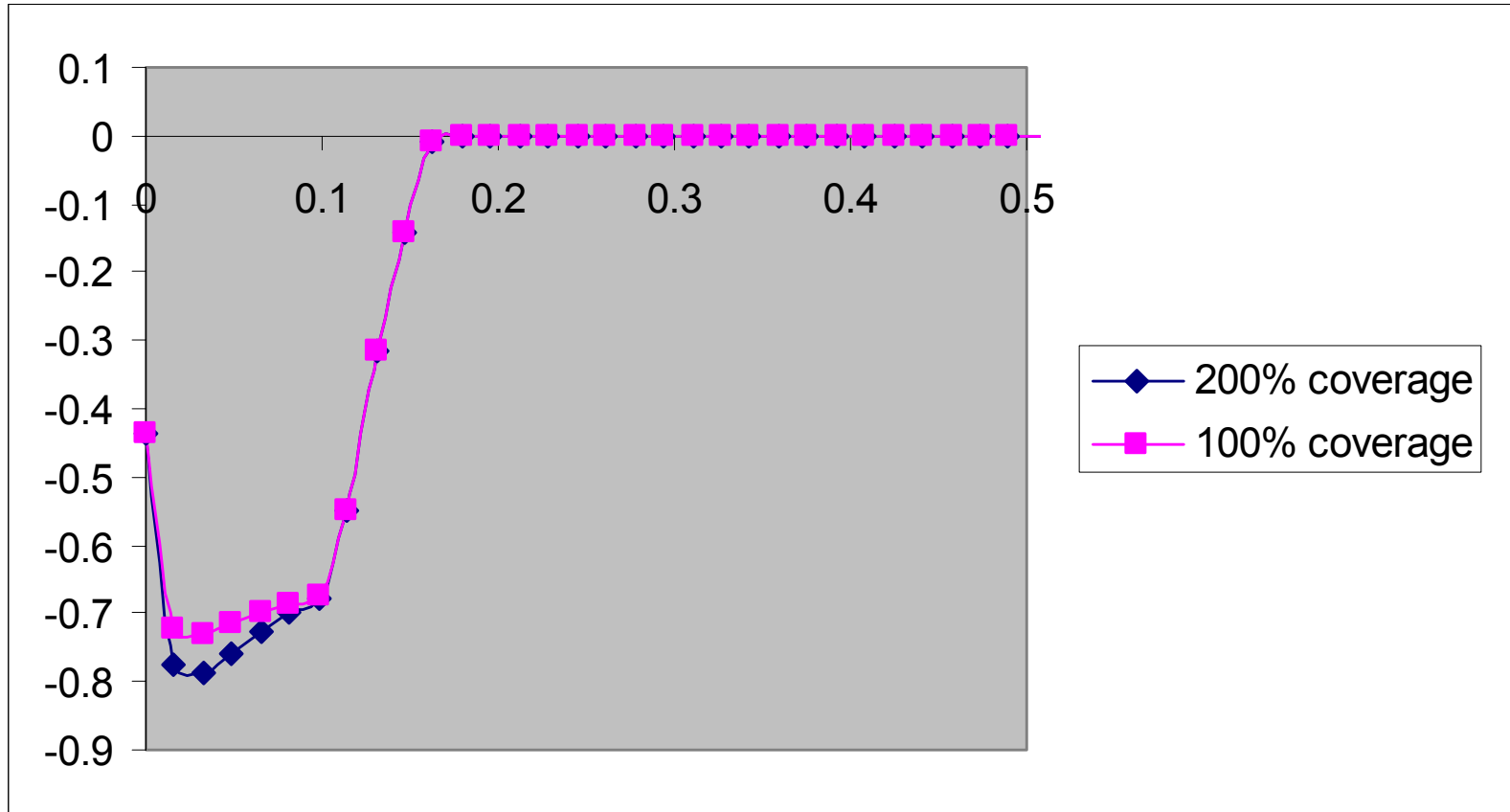
$R=0.55\text{mm}$, $E=200\text{GPa}$, $\nu=0.3$, $\rho=7800\text{kg/m}^3$, $V=36.58\text{m/s}$
 $\sigma_s=1.27\text{GPa}$, $\sigma_b=1.54\text{GPa}$, $\varepsilon_b=0.045$

The effect of the coverage



$R=0.275\text{mm}$, $E=200\text{GPa}$, $\nu=0.3$, $\rho=7800\text{kg/m}^3$, $V=63.58\text{m/s}$
 $\sigma_s=1.27\text{GPa}$, $\sigma_b=1.54\text{GPa}$, $\varepsilon_b=0.045$

The effect of the coverage



$R=0.3\text{mm}$, $E=70\text{GPa}$, $\nu=0.33$, $V=30\text{m/s}$, $\rho=2700\text{kg/m}^3$

$\sigma_s=0.462\text{GPa}$, $\sigma_b=0.526\text{GPa}$, $\varepsilon_b=0.11$

7075 Aluminium

Conclusions on the analytical model for shot-peening with 200% coverage

- ▶ This theoretical model considers the influence of the main parameters of shot peening: velocity of the shot, diameter of the shot, and the material characteristics;
- ▶ This model can be easily extended to 300% or higher coverage;
- ▶ This model verifies that the residual stress field will reach a converged state after certain coverage;
- ▶ This model is very simple and fast; no additional empirical parameters are introduced.

Test Data

The following experimental data needed by UCI

- the residual stress levels, and material parameters;
- specimen types, and sizes;
- the distribution of the residual stresses due to rivet misfit;
- the distribution of the residual stresses due to cold working;
- the distribution of the residual stresses due to shot-peening with 200% coverage (including shot velocity, shot radius).

to validate the analytical models.

Wichita state data

Tensile test properties

1. 7050 T7451 Al, 7 specimens
2. 7075 T7351 Al, 7 specimens

Residual stresses tests

Shot-peening

1. 7050 T7451 Al, 100% coverage, 1 specimen
2. 7075 T7351 Al, 200% coverage, 1 specimen

Up to December 3 2004

Average Tensile Properties

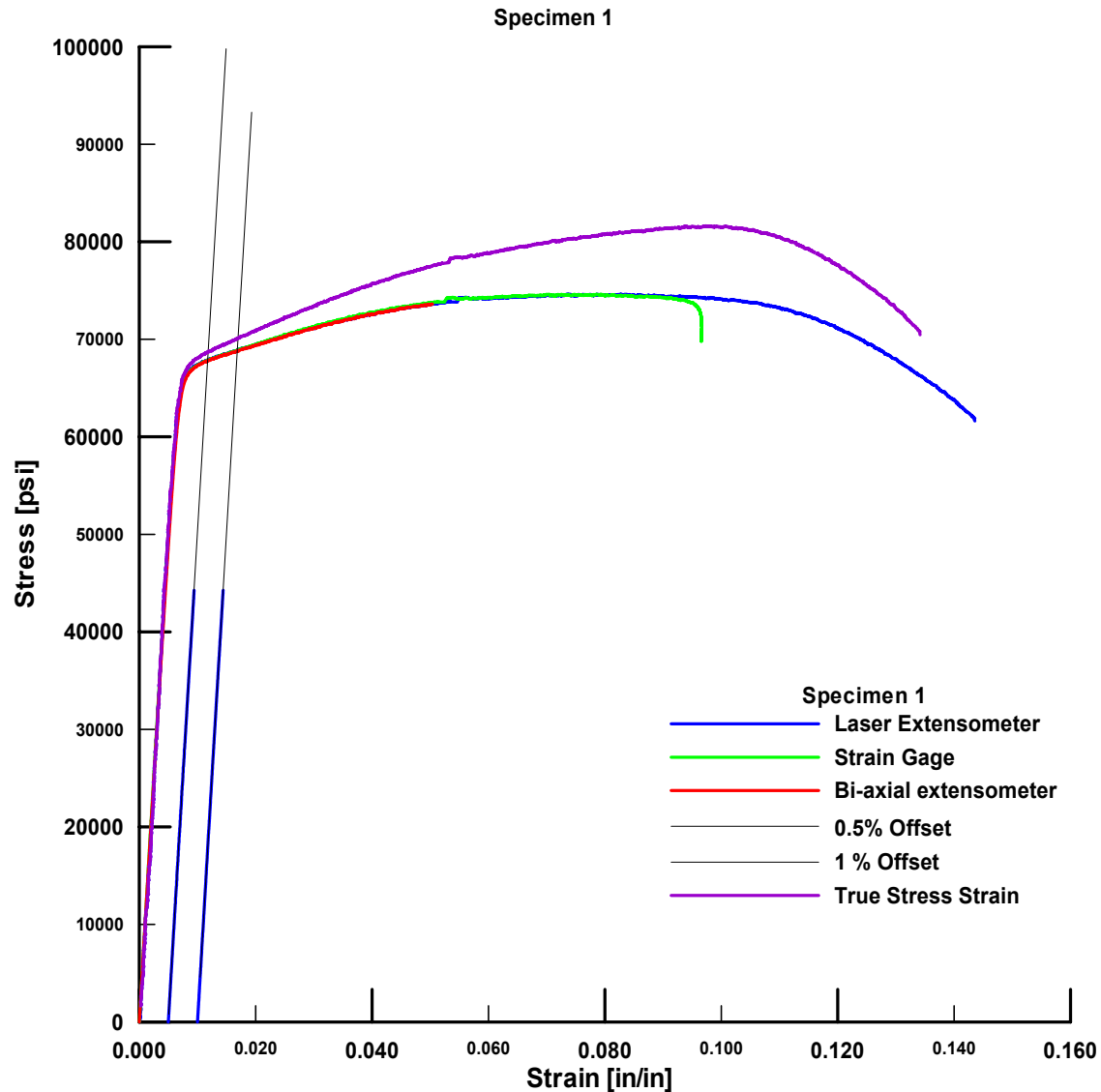
Aluminium 7050-T7451 (0.25" thick sheet) (Wichita state test data)

	σ_{ultimate} Ksi	σ_{Failure} Ksi	E Msi	ν	ϵ %	0.5% σ_{Yield} Ksi	1% σ_{Yield} Ksi
Average	75.07	61.56	10.10	0.336	13.697	68.29	69.17
Std. Dev	0.319	0.568	0.047	0.009	0.7440	0.258	0.300
CoV	0.425	0.923	0.461	2.605	5.432	0.377	0.434

The measured density is 2.76 g/cm³

Stress- Strain Curve

Specimen 1 Al-7050-T7451 (Wichita state test)



Experiment (Wichita state)

7050 –T7451 alloys

Shot-peening parameters

Measured intensity – $0.077 \sim 0.078A$

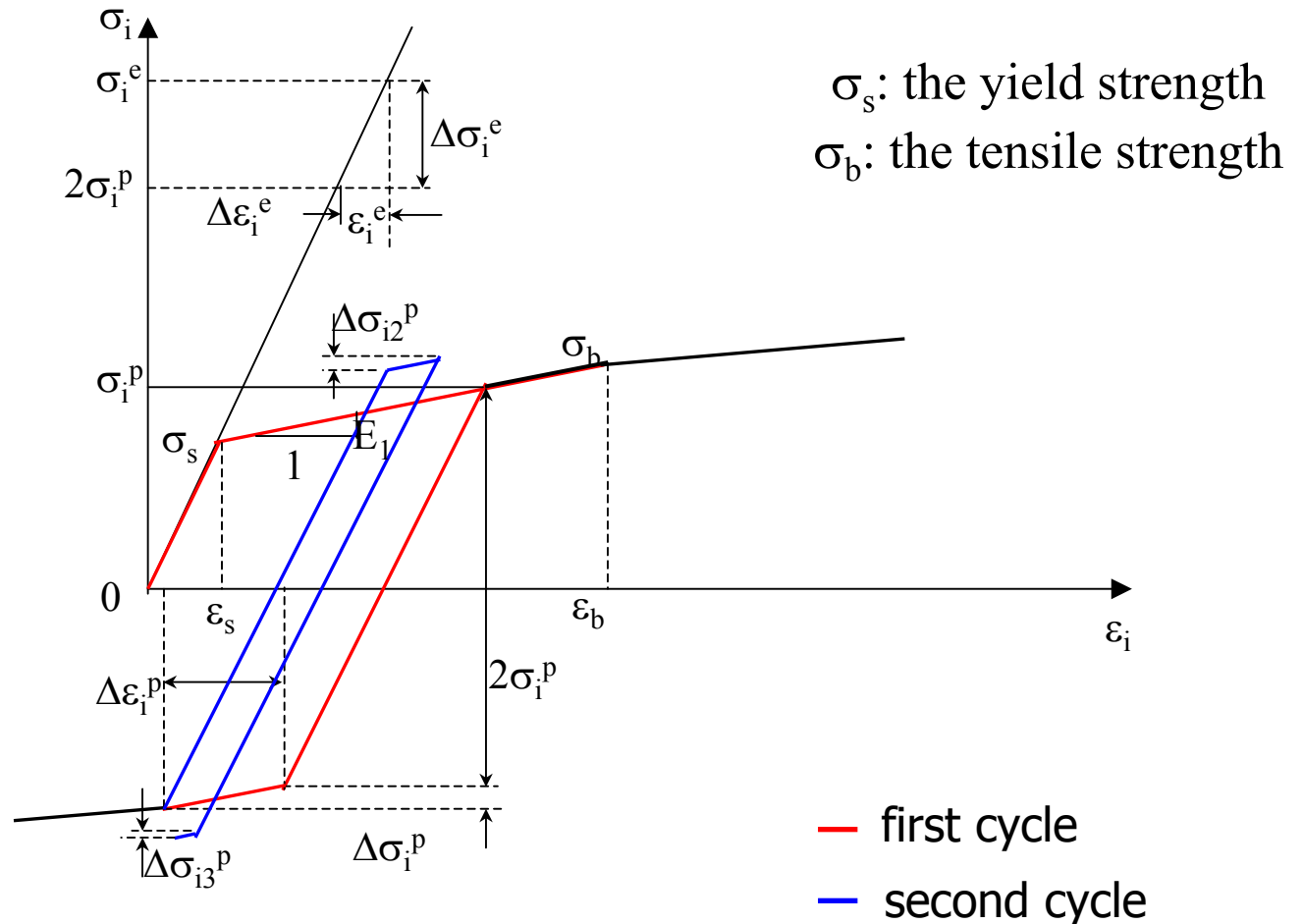
100% and 200% coverage's

Shot diameter- 230R (0.023in)

Cast Steel Shots

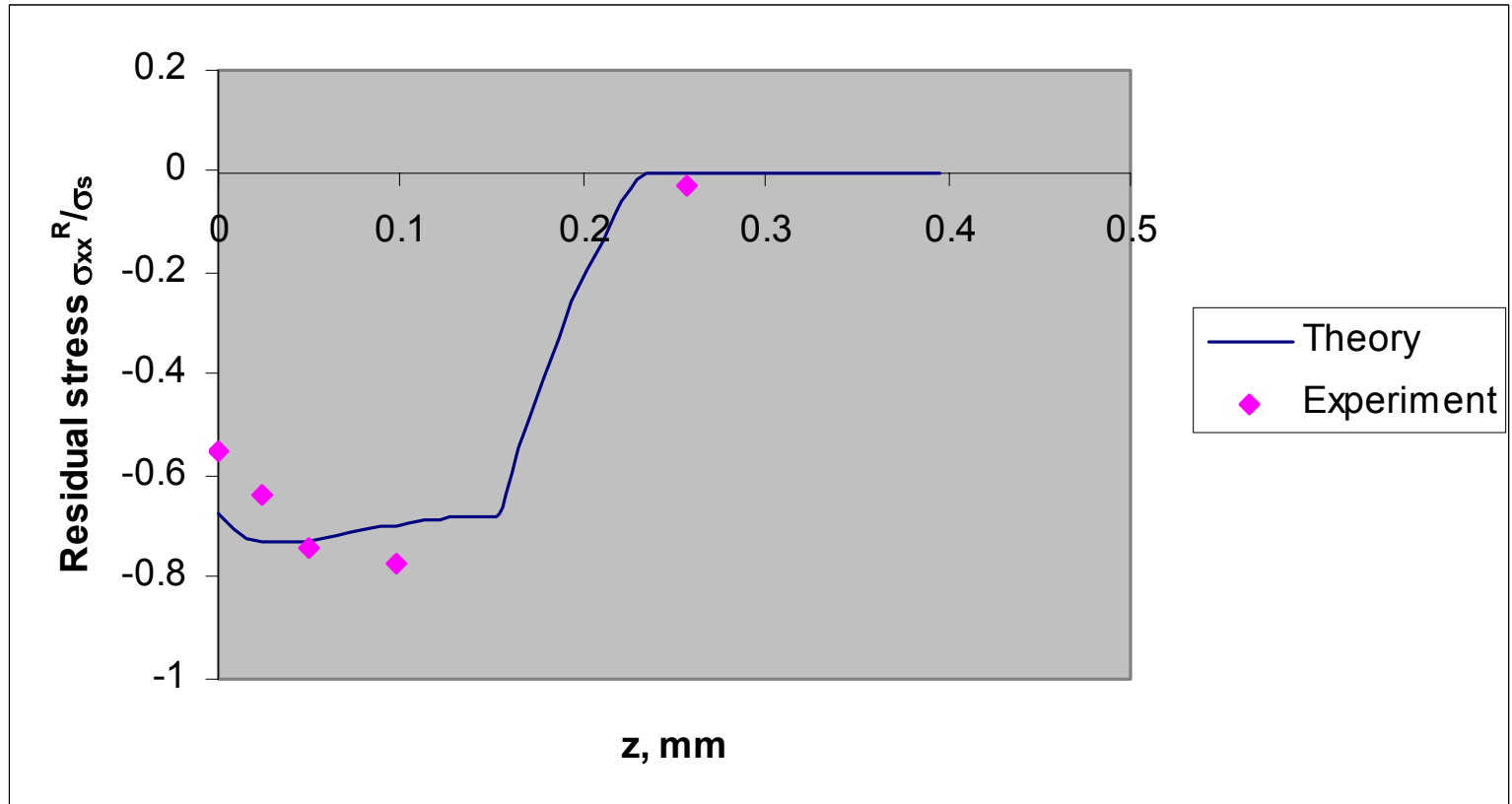
Only one specimen

Schematic diagram for calculating residual stress-200% Coverage



Isotropic hardening

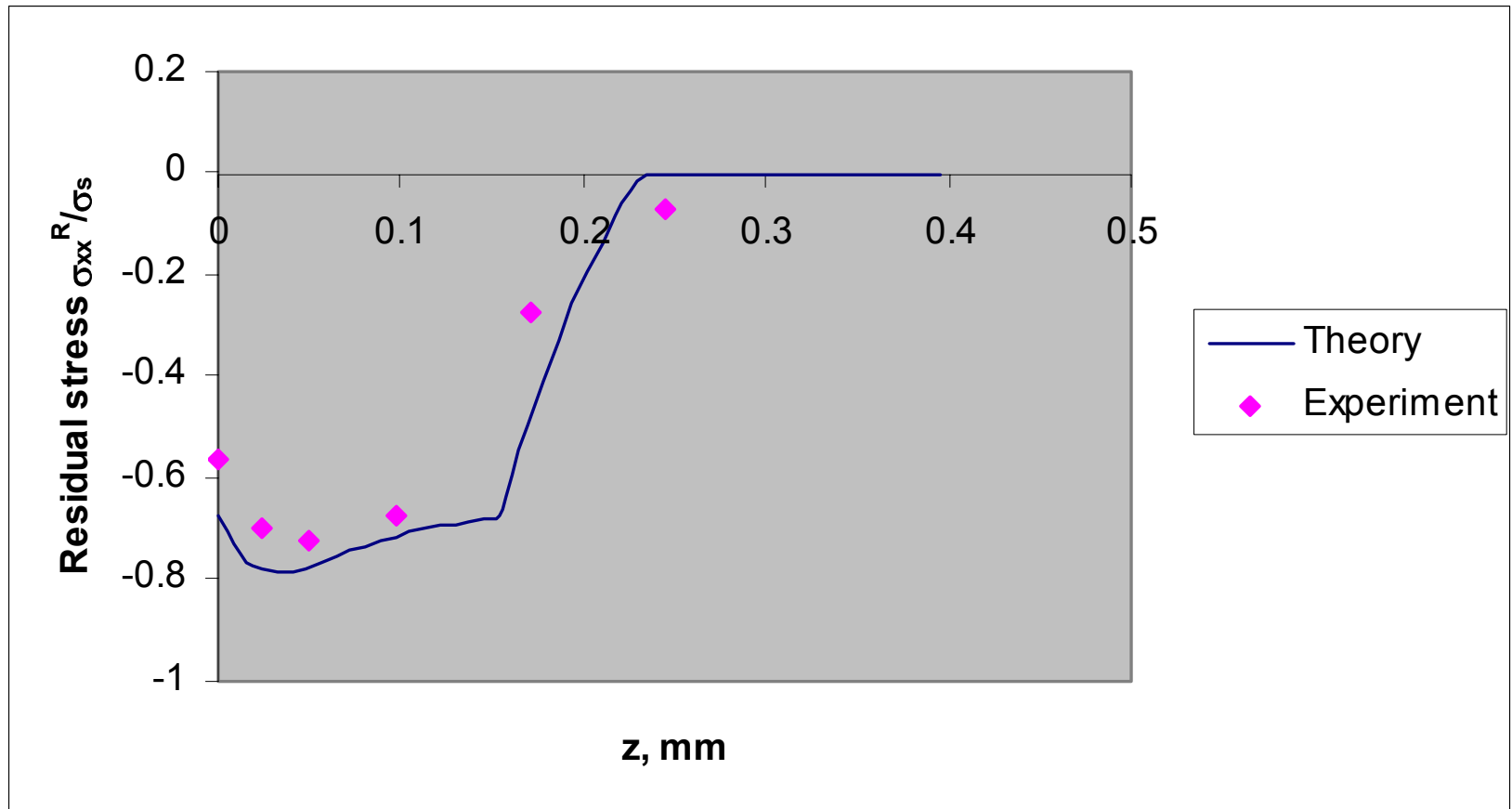
Comparison of the experimental and theoretical results



100% coverage

7050 –T7451 alloys

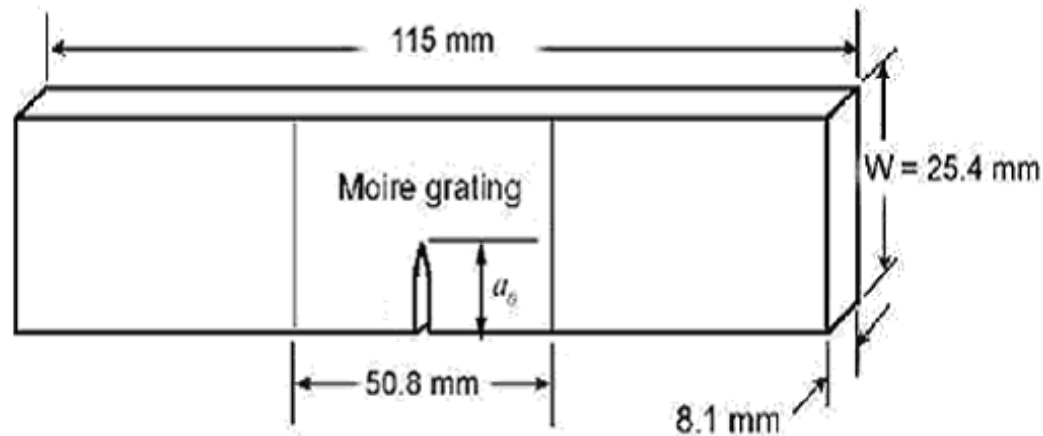
Comparison of the experimental and theoretical results



200% coverage

7050 –T7451 alloys

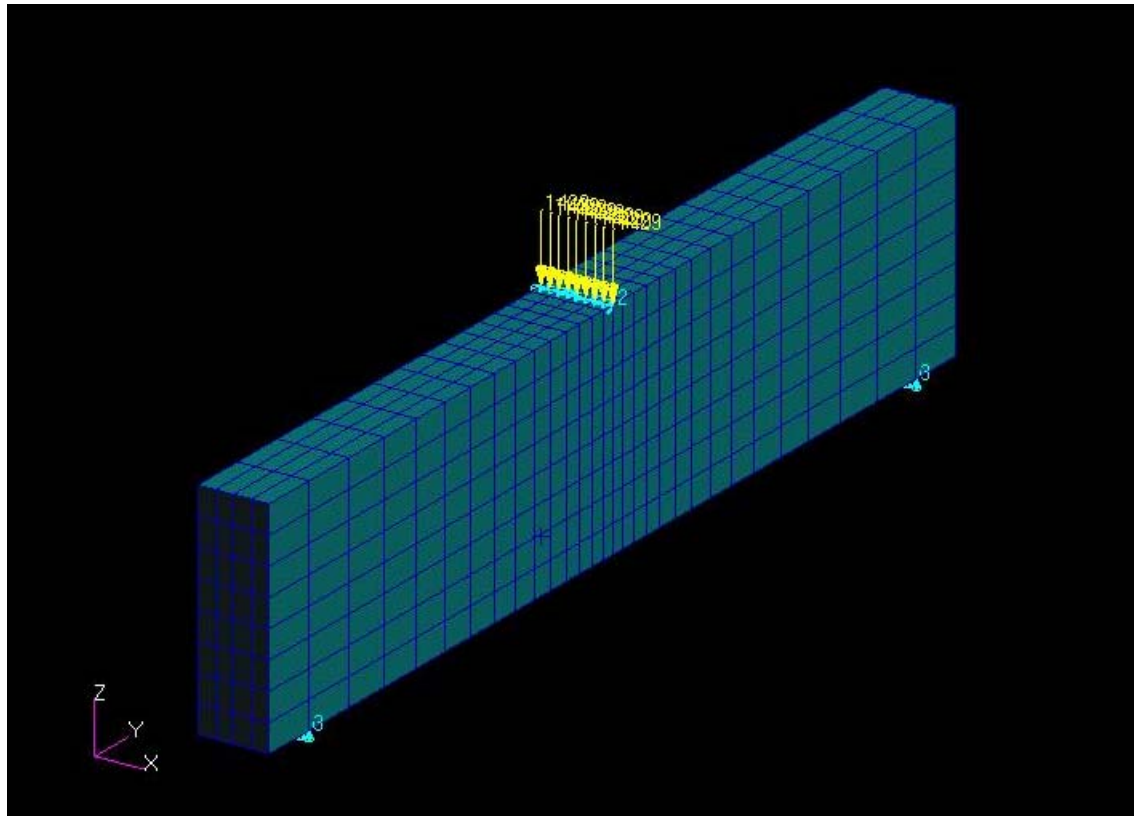
Fatigue Crack Growth in Aluminum Three-Point Bend Specimens



Single Edged Notch Bend (SENB) Specimen

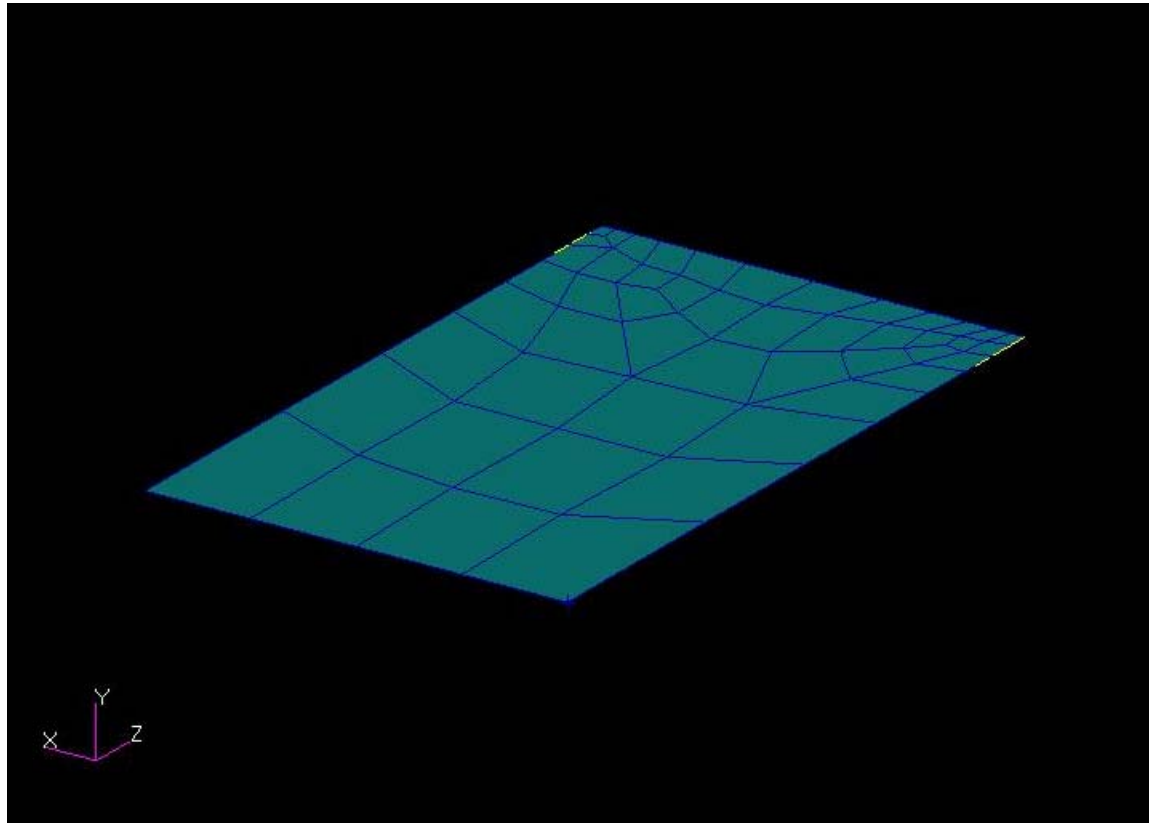
Material 2024-T351 Al, $a_0 = 13 \text{ mm}$

Fatigue Crack Growth in Aluminum Three-Point Bend Specimens



Global FEM model, 960 elements (Hexahedral 20)

Fatigue Crack Growth in Aluminum Three-Point Bend Specimens

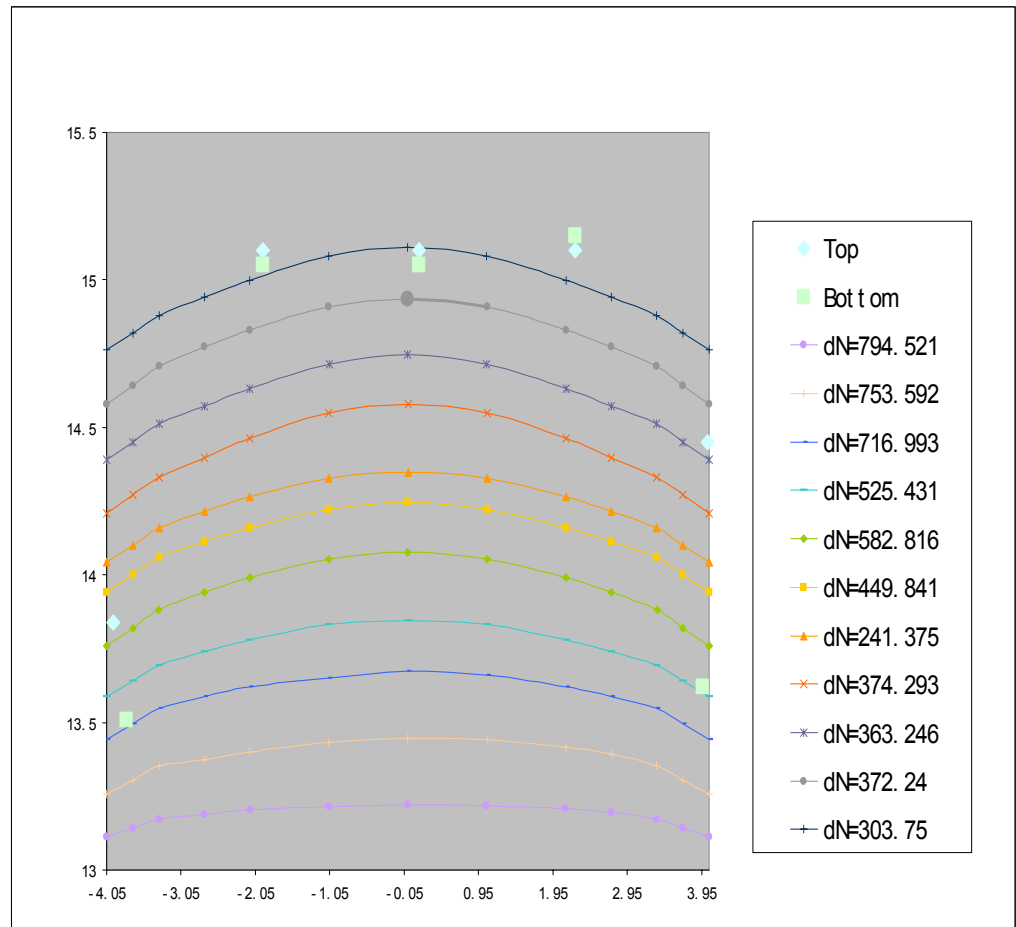


BEM model in the crack plane,
12 Elements along crack front (Quadrangular 8)

Fatigue Crack Growth in Aluminum Three-Point Bend Specimens

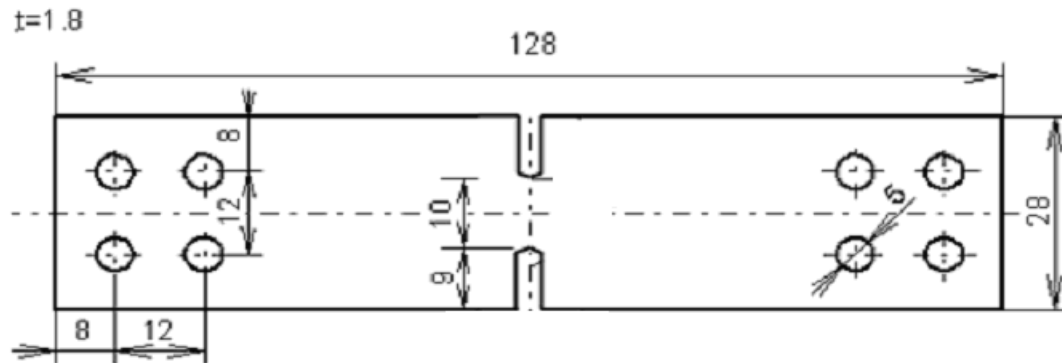
Test A2			
P_{\max} (kN)	P_{\min} (kN)	Cycles	
1.725	0.173	4963	
Top		Bottom	
X (mm)	Y (mm)	X (mm)	Y (mm)
0.84	0.09	0.51	0.27
2.10	2.10	2.05	2.10
2.10	4.20	2.05	4.20
2.10	6.30	2.15	6.30
1.45	8.08	0.62	8.01

A2
Numerical N = 5480
Experimental N = 4963



FATIGUE CRACK GROWTH IN ALUMINUM

double-edged crack (UW Test Data)

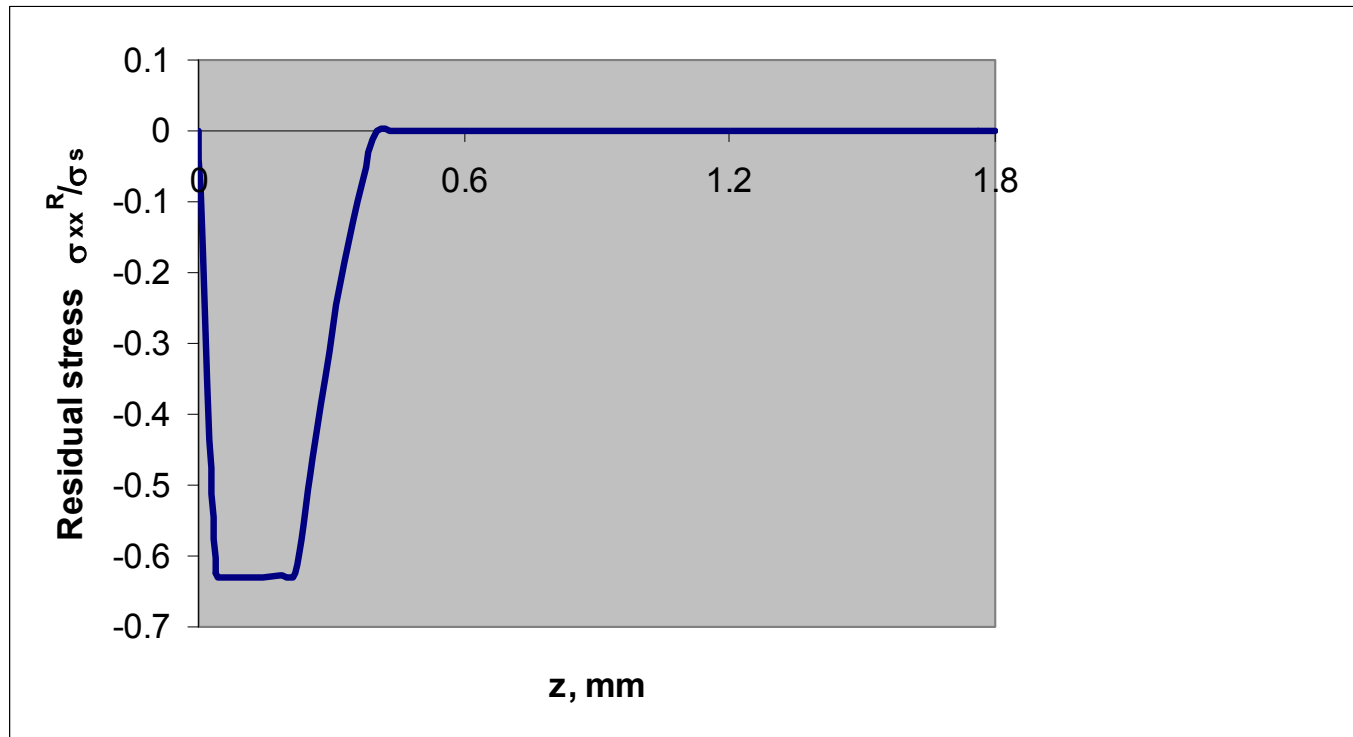


Tension, 400lb, Material 7075-T7351 Al

Shot Peening

Intensity 0.017, shot size 230-280, coverage 100%

Distribution of the residual stress due to shot-peening

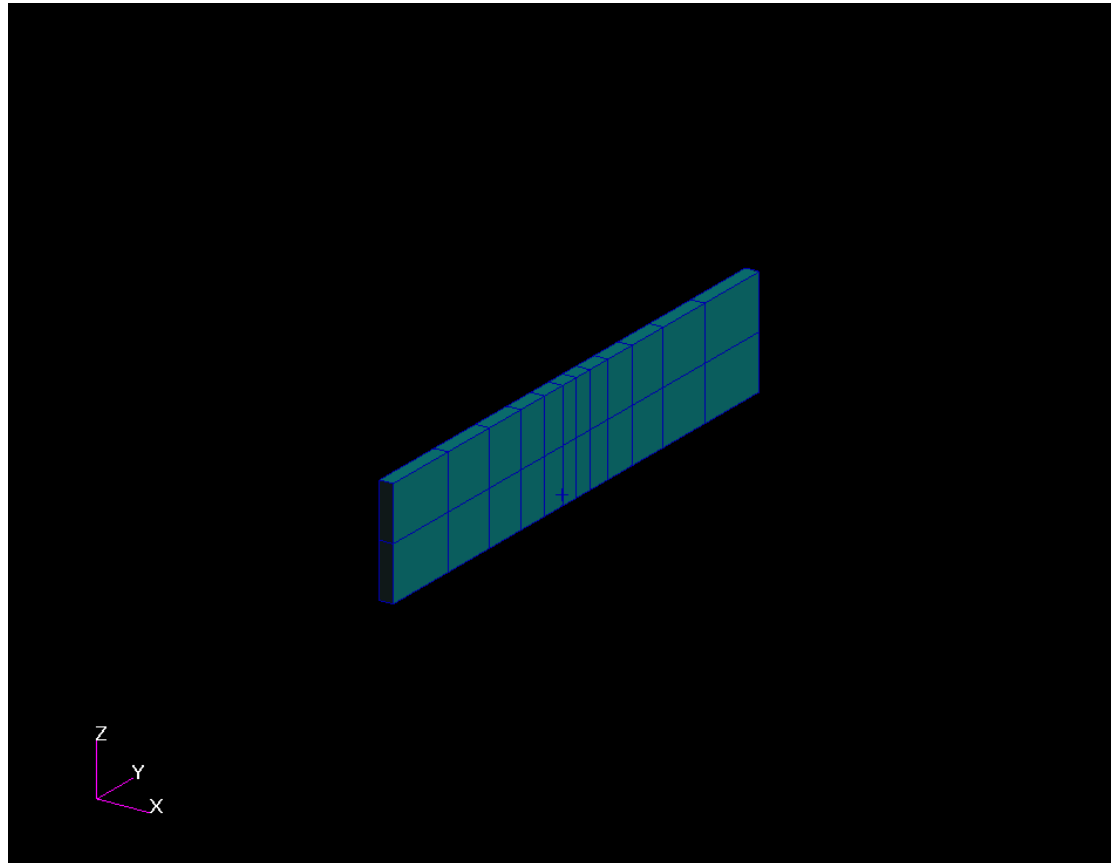


Shot Peening

Intensity 0.017, shot size 230-280, coverage 1.0

FATIGUE CRACK GROWTH IN ALUMINUM

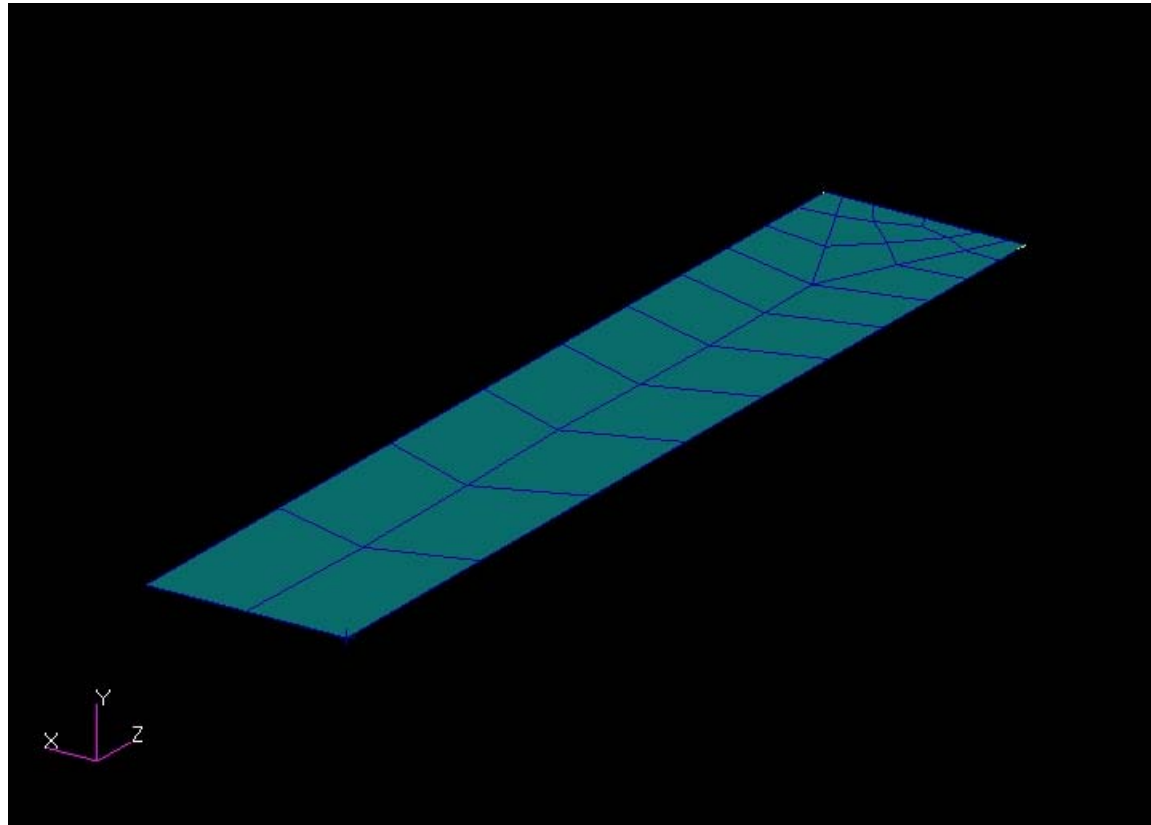
double-edged crack



Global FEM model, 24 elements (Hexahedral 20)

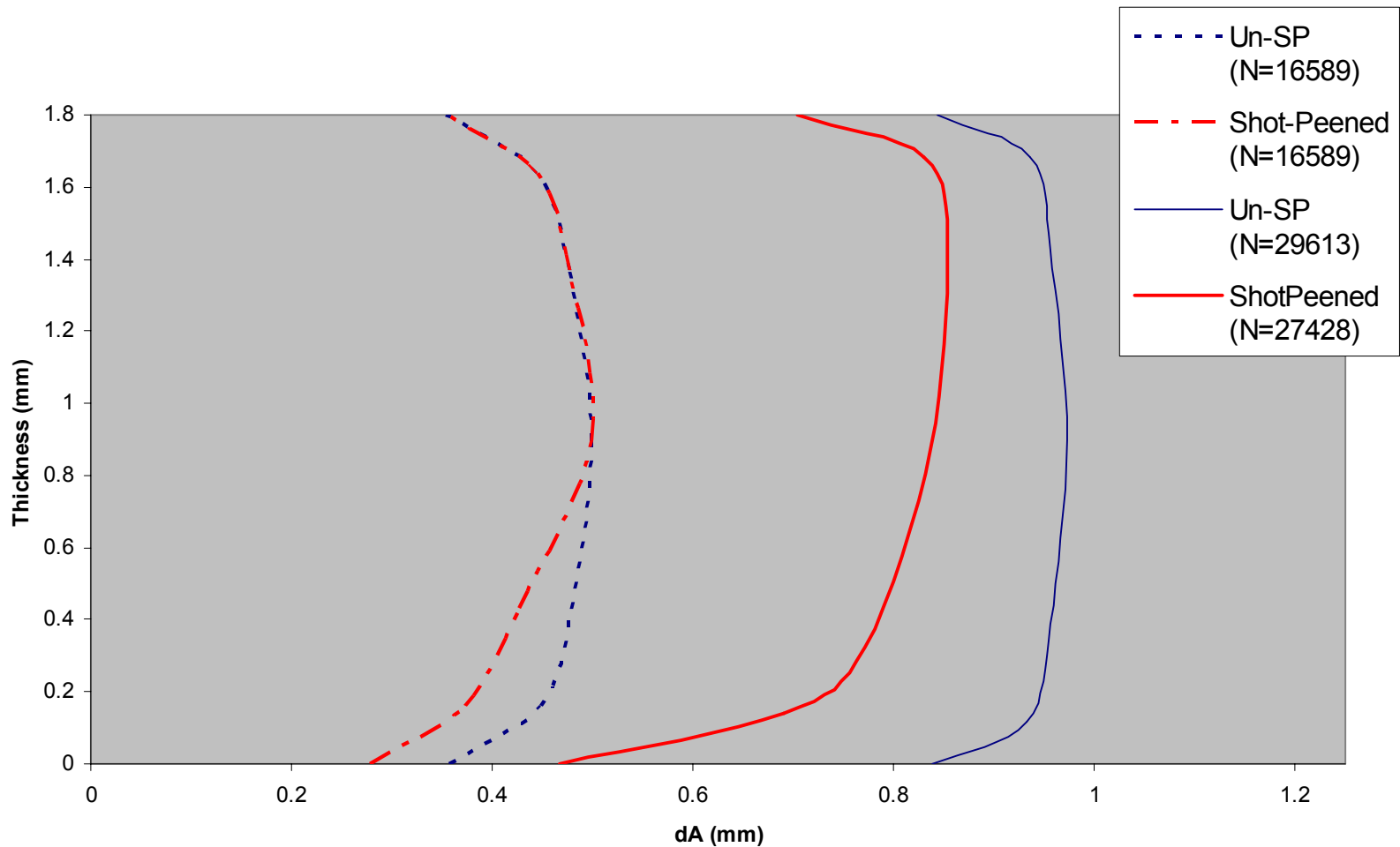
FATIGUE CRACK GROWTH IN ALUMINUM

double-edged crack



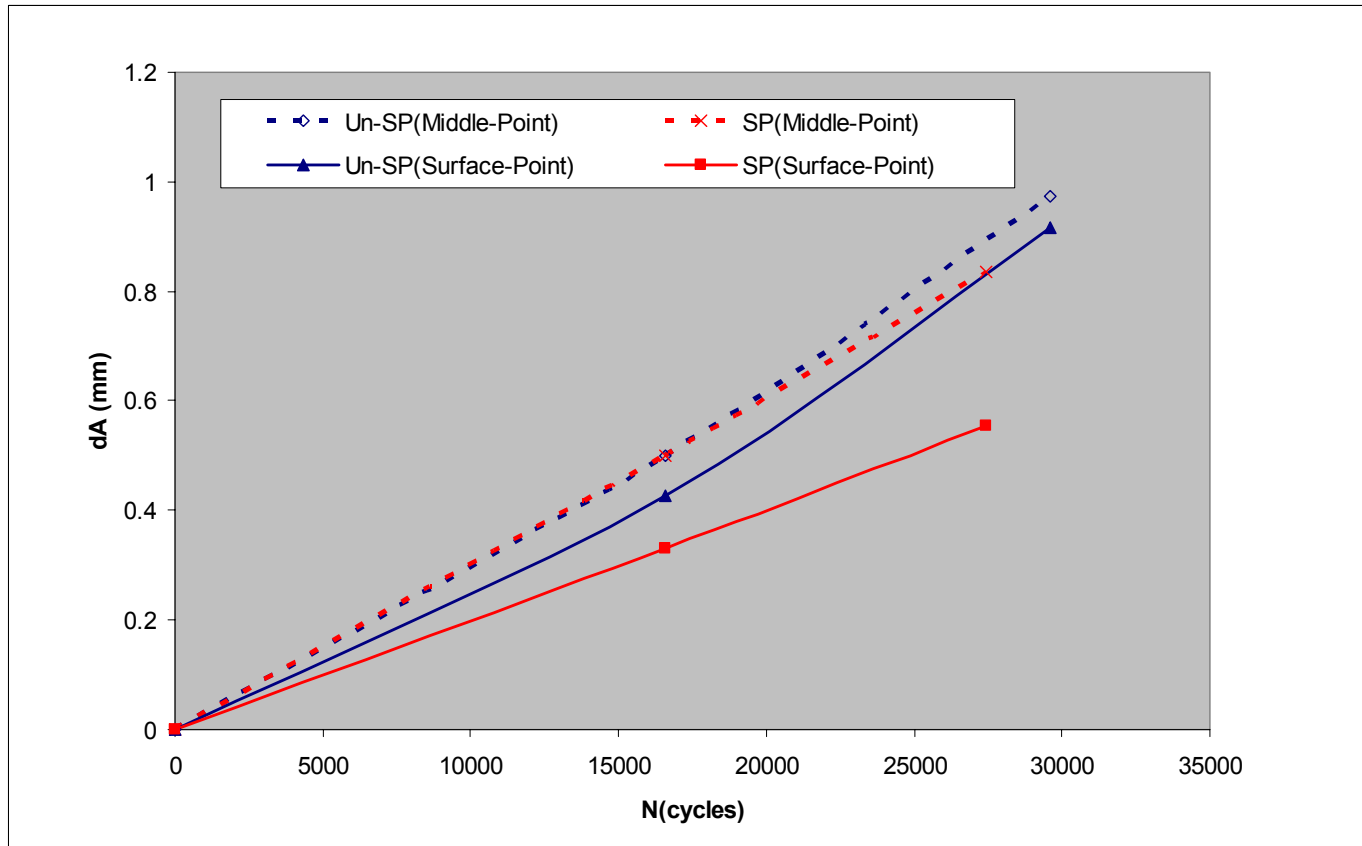
BEM model in the crack plane,
6 Elements along crack front (Quadrangular 8)

Crack Profiles



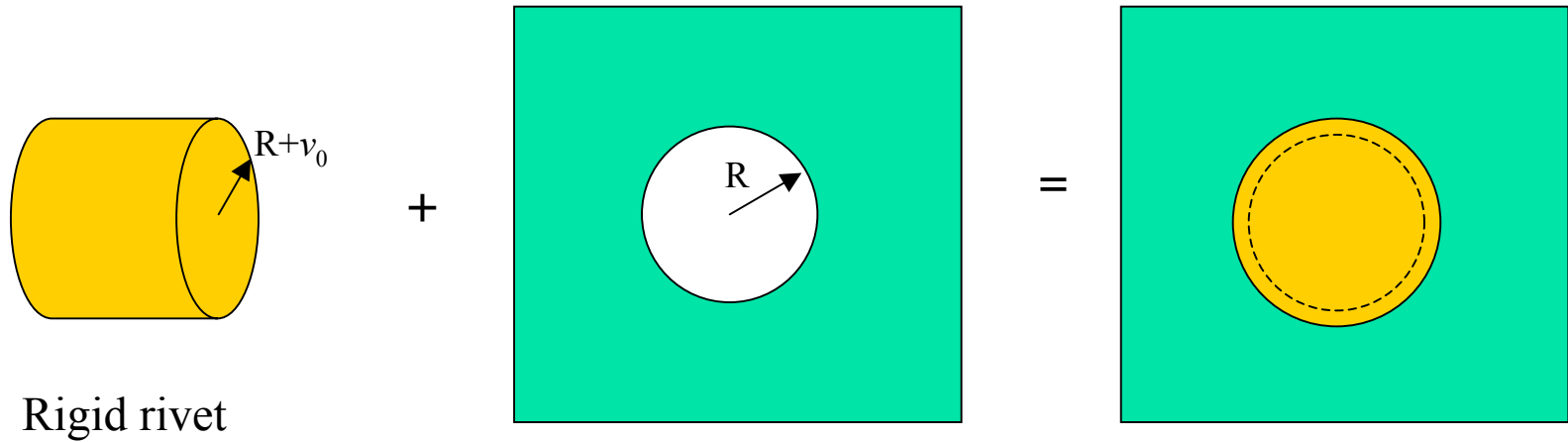
Numerical results

The Effect of the Shot-Peening



Numerical results

Effect of Residual Stresses in the Fastener Hole



For a single hole in an infinite sheet:
The radial pressure p_0 on the hole surface

$$p_0 = (2\mu) \frac{v_0}{R} = k_0 \frac{v_0}{R}$$

k_0 is the “stiffness” of the hole in an infinite sheet

Fatigue Test Data

The following experimental data should be provided to UCI

- the residual stress levels, specimen types, crack sizes, and material parameters;
- load spectrum;
- fatigue life curves: $a \sim N$, and $da/dN \sim \Delta K$;
- fatigue crack profiles of the specimens.

Effect of plastic deformation due to cold-working

Assuming that the plastic deformation is caused solely by cold-working of the fastener-hole; and that the applied far-field hoop stress does not produce any plastic deformation. The material is regarded to be elastic-perfectly-plastic.

For the elastic deformation, the stress-field near the hole

$$\sigma_{rr} = -p_0 \left(\frac{R}{r} \right)^2 \qquad \sigma_{\theta\theta} = p_0 \left(\frac{R}{r} \right)^2$$

r : the distance from the center of the hole.

R : the radius of the hole.

p_0 : the radial pressure applied on the hole surface

Effect of plastic deformation due to cold-working

As the pressure p_0 is increased, the material near the hole begins to deform plastically.

In the plastic region $R \leq r \leq r_y$ the stresses are given by

$$\sigma_{rr} = -p_0 + \sigma_{ys} \ln\left(\frac{r}{R}\right) \quad R \leq r \leq r_y$$

$$\sigma_{\theta\theta} = \sigma_{ys} - p_0 + \sigma_{ys} \ln\left(\frac{r}{R}\right) \quad R \leq r \leq r_y$$

Tresca yield condition are used.

σ_{ys} is the yield-strength of the material and r_y is the radius of the plastic-region

Effect of plastic deformation due to cold-working

In the elastic region

$$\sigma_{rr} = -\frac{\sigma_{ys}}{2} \left(\frac{r_y}{r} \right)^2 \quad r > r_y$$

$$\sigma_{\theta\theta} = \frac{\sigma_{ys}}{2} \left(\frac{r_y}{r} \right)^2 \quad r > r_y$$

r_y is determined by

$$\frac{r_y}{R} = e^{\left(\frac{p_0}{\sigma_{ys}} - \frac{1}{2} \right)}$$

Effect of plastic deformation due to cold-working

The residual stress-field can be obtained by subtracting the elastic solution from the plastic solution

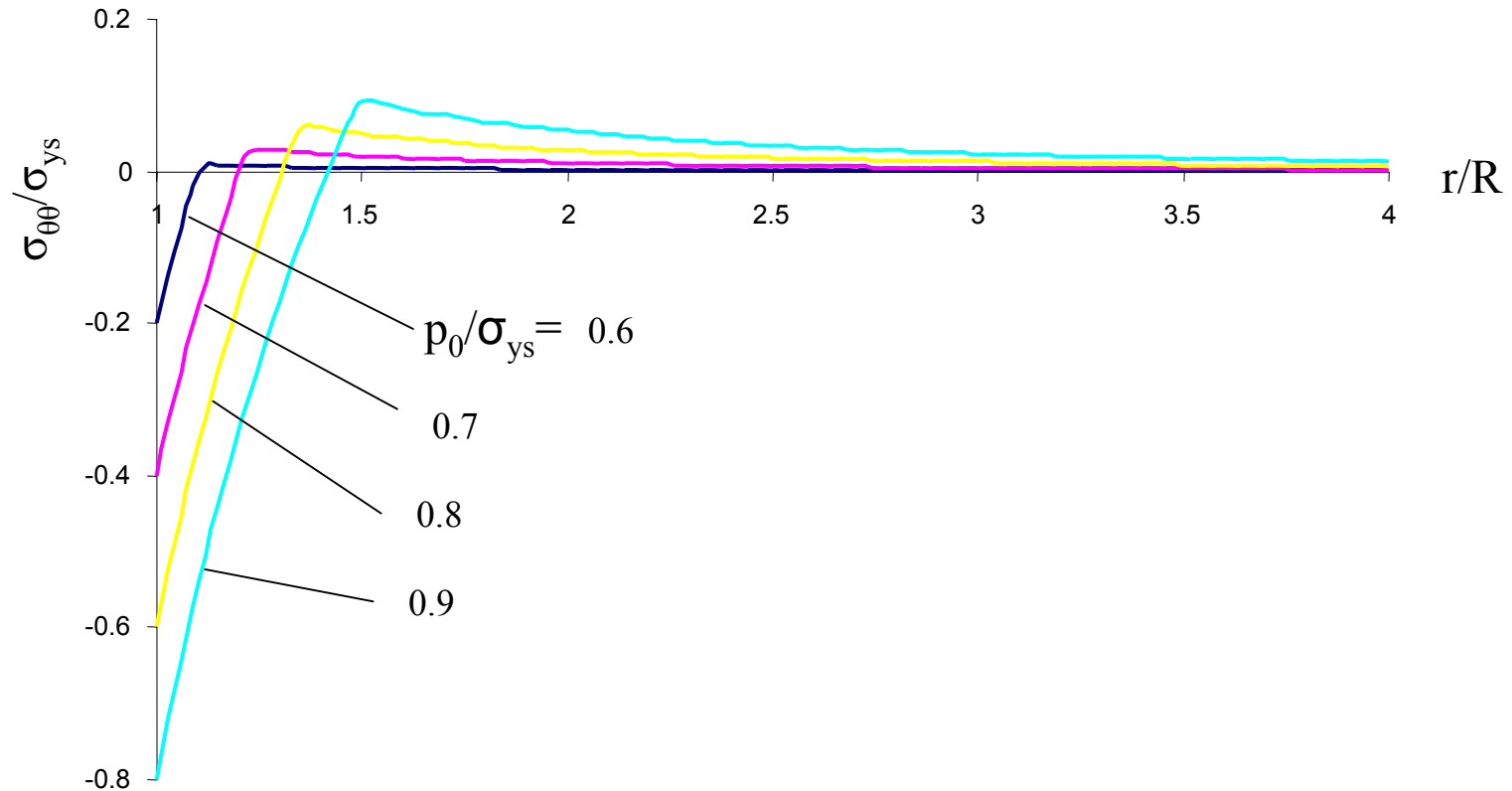
$$\sigma_{rr} = -p_0 + \sigma_{ys} \ln\left(\frac{r}{R}\right) + p_0 \left(\frac{R}{r}\right)^2 \quad R \leq r \leq r_y$$

$$\sigma_{\theta\theta} = \sigma_{ys} - p_0 + \sigma_{ys} \ln\left(\frac{r}{R}\right) - p_0 \left(\frac{R}{r}\right)^2 \quad R \leq r \leq r_y$$

$$\sigma_{rr} = -\frac{\sigma_{ys}}{2} \left(\frac{r_y}{r}\right)^2 + p_0 \left(\frac{R}{r}\right)^2 \quad r > r_y$$

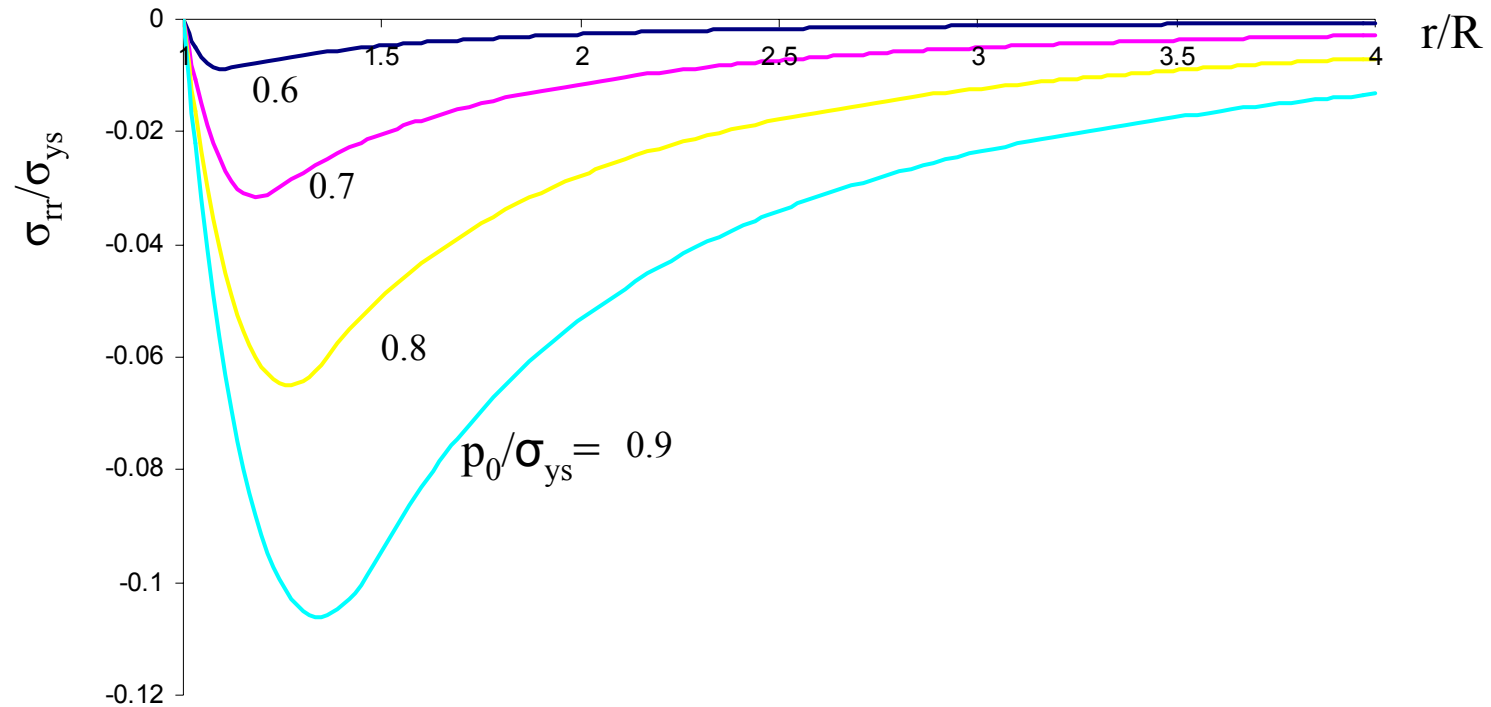
$$\sigma_{\theta\theta} = \frac{\sigma_{ys}}{2} \left(\frac{r_y}{r}\right)^2 - p_0 \left(\frac{R}{r}\right)^2 \quad r > r_y$$

Effect of plastic deformation due to cold-working



Residual stress $\sigma_{\theta\theta}$ along a radial line from the center of the hole, due to cold-working induced plasticity

Effect of plastic deformation due to cold-working



Residual stress σ_{rr} along a radial line from the center of the hole, due to cold-working induced plasticity

Modeling fatigue crack growth - Plastic Zone Model

The effects of the shot-peening, cold-working on the crack growth is same as the effect of the overload: the residual stress field impedes the crack propagation. This model is based on the shape of the plastic zone.

Accounting the 3-D effect by assuming that

$$\sigma_3 = T_z (\sigma_2 + \sigma_1)$$

T_z is the 3D constraint factor. For plane stress $T_z = 0$

For plane strain $T_z = \nu$ ν is the Poisson's ratio.

T_z should vary along the thickness, i.e. is a function of z . In the center of the specimen, $T_z = \nu$; on the surface, $T_z = 0$.

Modeling fatigue crack growth - Plastic Zone Model

The principal stresses for the Mode I crack can be written as

$$\sigma_1 = \frac{K_I}{\sqrt{2\pi r}} \cos\left(\frac{\theta}{2}\right) \left[1 + \sin\left(\frac{\theta}{2}\right)\right]$$

$$\sigma_2 = \frac{K_I}{\sqrt{2\pi r}} \cos\left(\frac{\theta}{2}\right) \left[1 - \sin\left(\frac{\theta}{2}\right)\right]$$

and

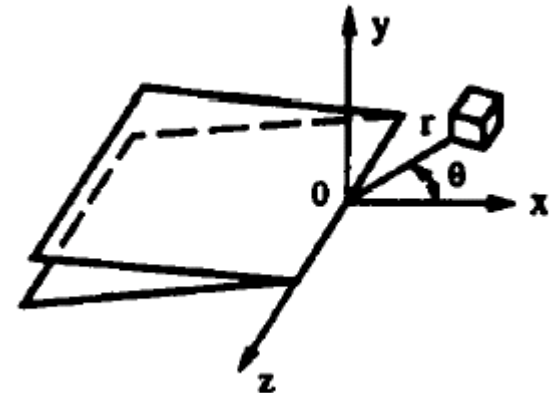
$$\sigma_3 = \frac{2T_z K_I}{\sqrt{2\pi r}} \cos\left(\frac{\theta}{2}\right)$$

T_z can be approximately related to the plastic constrain factor α in NASGRO model (averaging T_z along the thickness)

$$T_z = \frac{\nu(\alpha - 1)}{2}$$

$\alpha = 1$ plane stress

$\alpha = 3$ plane strain



Modeling fatigue crack growth - Plastic Zone Model

Consider the von Mises equation the effective stress is

$$\sigma_e = \frac{1}{\sqrt{2}} \left[(\sigma_1 - \sigma_2)^2 + (\sigma_1 - \sigma_3)^2 + (\sigma_2 - \sigma_3)^2 \right]^{\frac{1}{2}}$$

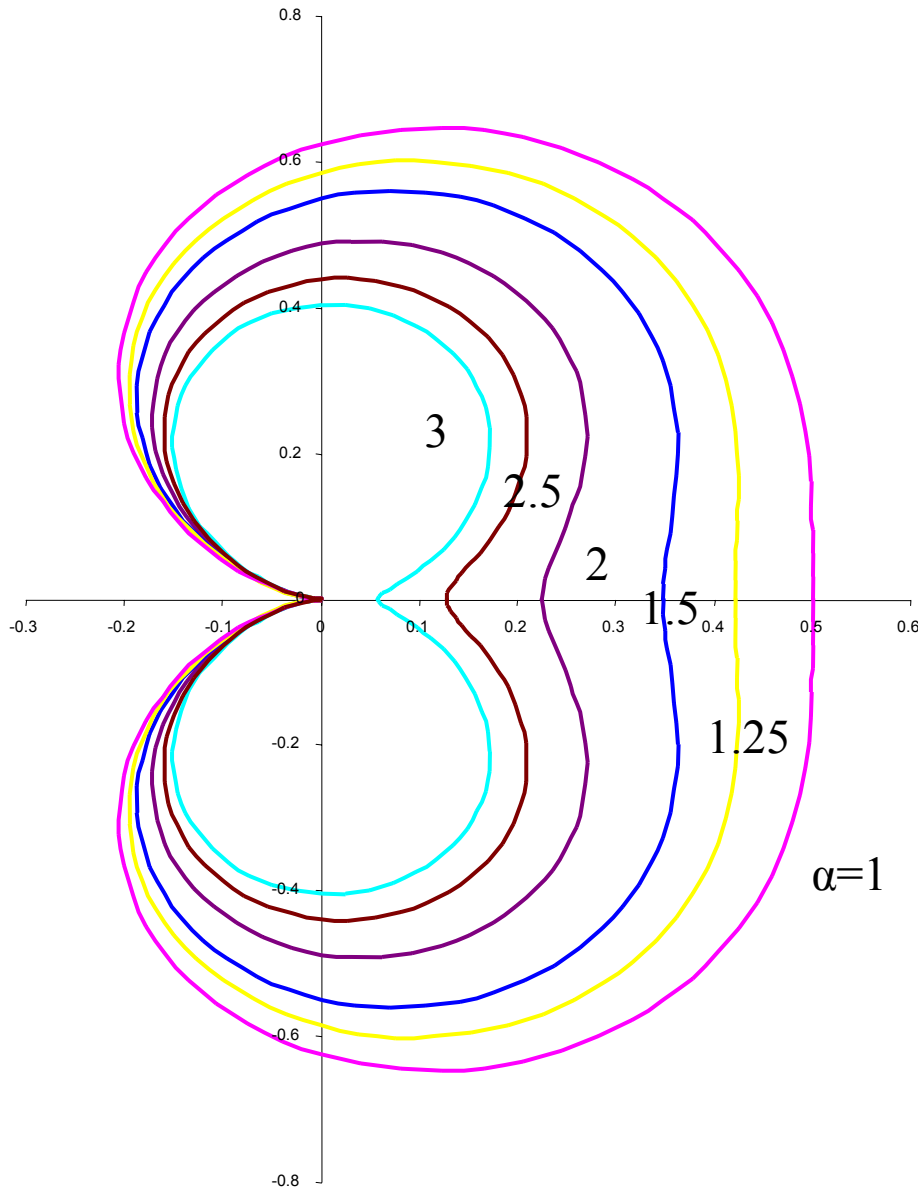
According to the von Mises criterion, yielding occurs when

$\sigma_e = \sigma_0$, the uniaxial yield strength

The Mode I plastic zone radius can be estimate as

$$r(\theta) = \frac{1}{4\pi} \left(\frac{K_I}{\sigma_0} \right)^2 \left[(1 - 2T_z)^2 (1 + \cos \theta) + \frac{3}{2} \sin^2 \theta \right]$$

Crack tip plastic zone shapes



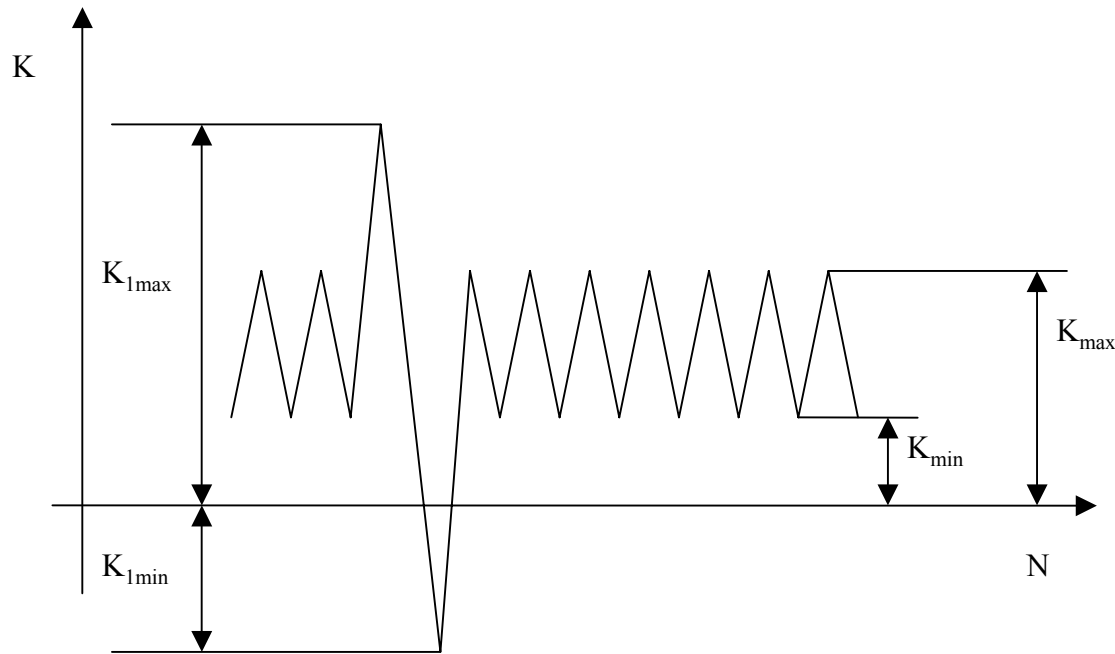
For plane stress $\alpha=1$

For plane strain $\alpha=3$

The crack tip plastic zone shapes
under different 3D constraint

Plastic Zone model

Consider a structure subjected to a cyclic load with an overload.



$$R_1 = \frac{K_{lmin}}{K_{lmax}}$$

$$R = \frac{K_{min}}{K_{max}}$$

For the residual stress field due to the shot-peening or cold-working, $R_1=0$.

Plastic Zone model

After the application of the overload, the plastic zones formed at the crack tip are

The plastic zone of overload

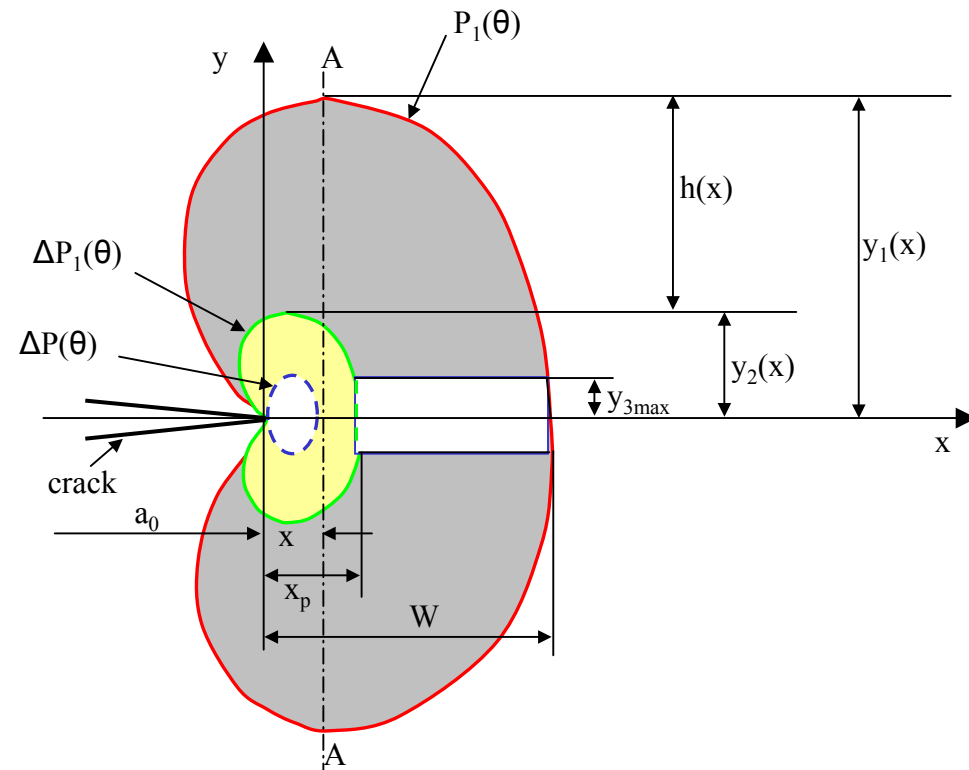
$$P_1(\theta) = \frac{1}{4\pi} \left(\frac{K_{1max}}{\sigma_0} \right)^2 f(\theta)$$

The cyclic plastic zone of overload
(isotropic hardening)

$$\Delta P_1(\theta) = \frac{\beta}{4\pi} \left(\frac{(1 - R_1)K_{1max}}{2\sigma_0} \right)^2 f(\theta)$$

The baseline cyclic plastic zone

$$\Delta P(\theta) = \frac{\beta}{4\pi} \left(\frac{(1 - R)K_{max}}{2\sigma_0} \right)^2 f(\theta)$$



β is the cyclic plastic zone size factor and depends on R

Plastic Zone model

During the loading half cycle, the crack will be opened only when the load is great enough to make the CTOD equal to the compressive deformation during the unloading half cycle

$$\text{CTOD} = 2\delta \quad \delta = \lambda \Delta l$$

Δl is the stretch of the material element. For an element just behind the crack tip, it can be calculated as

$$\Delta l = \int_{y_2}^{y_1} \varepsilon_p dy$$

Approximately

$$\varepsilon_p = m \frac{\sigma_0}{E}$$

m is a magnification factor depending on x , and is assumed to be proportional to the height $h(x)$ within the shade area,

$$m = m_0 \frac{h(x)}{w}$$

m_0 is a constant, w is the length of the overload affected zone

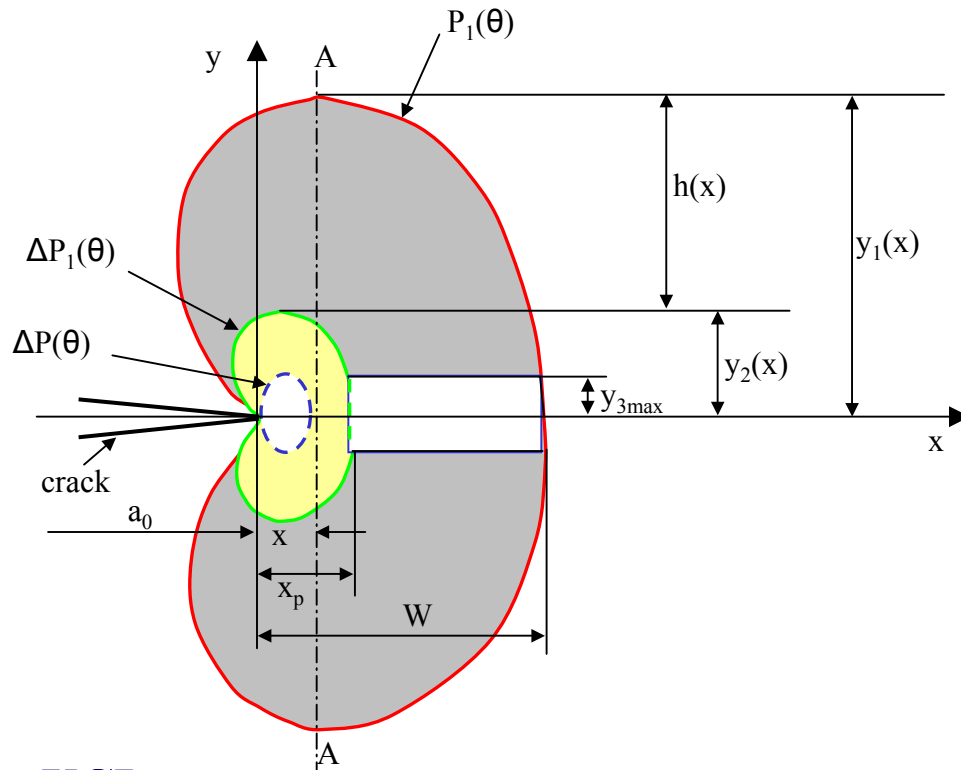
Plastic Zone model

From the figure,

$$h(x) = y_1(x) - y_2(x) \quad \text{for } x < x_p$$

$$h(x) = y_1(x) - y_{3_{\max}}(x) \quad \text{for } x \geq x_p$$

Here



$$y_1(x) = \frac{1}{4\pi} \left(\frac{K_{1\max}}{\sigma_0} \right)^2 \bar{y}_1(x)$$

$$y_2(x) = \frac{\beta}{4\pi} \left(\frac{(1-R_1)K_{1\max}}{2\sigma_0} \right)^2 \bar{y}_2(x)$$

$$y_{3\max} = \frac{\beta}{4\pi} \left(\frac{(1-R)K_{\max}}{2\sigma_0} \right)^2 \bar{y}_{\max}$$

Plastic Zone model

The crack tip opening displacement can be expressed as

$$CTOD = k \frac{K_{1max}^2}{E\sigma_0}$$

and

$$\delta = \frac{\lambda m_0 \sigma_0}{Ew} h^2(x)$$

The crack opening SIF

$$K_{op}(x) = M_0 \left[\bar{y}_1(x) - \frac{\beta(1-R_1)^2}{4} \bar{y}_2(x) \right] R_m K_{max} \quad \text{for } x < x_p$$

$$K_{op}(x) = M_0 \left[\bar{y}_1(x) - \frac{\beta(1-R)^2}{4R_m^2} \bar{y}_{max} \right] R_m K_{max} \quad \text{for } x \geq x_p$$

M_0 is an empirical material constant.

$$R_m = \frac{K_{1max}}{K_{max}}$$

Plastic Zone model

M_0 can be determined from known test data, or by means of the NASGRO model, under constant amplitude cyclic load, i.e. $R_m=1$.

This model

$$K_{op}(x) = M_0 \left[\bar{y}_1(x) - \frac{\beta(1-R)^2}{4R_m^2} \bar{y}_{\max} \right] K_{\max}$$

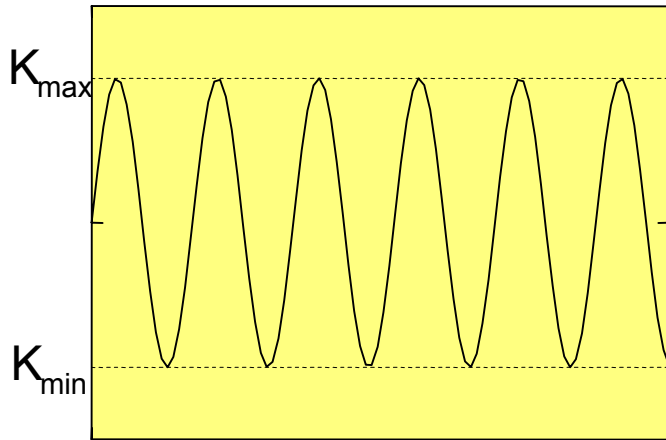
NASGRO

$$\frac{K_{op}}{K_{\max}} = \begin{cases} \max(R, A_0 + A_1 R + A_2 R^2 + A_3 R^3), & R \geq 0 \\ A_0 + A_1 R, & -2 \leq R < 0 \\ A_0 - 2A_1, & R < -2 \end{cases}$$

Equaling these two equations at $R=0$  M_0

Plastic Zone model

Fatigue crack growth rate



$$\frac{da}{dn} = C(\Delta K_{eff})^n$$

$$\Delta K = K_{max} - K_{min}$$

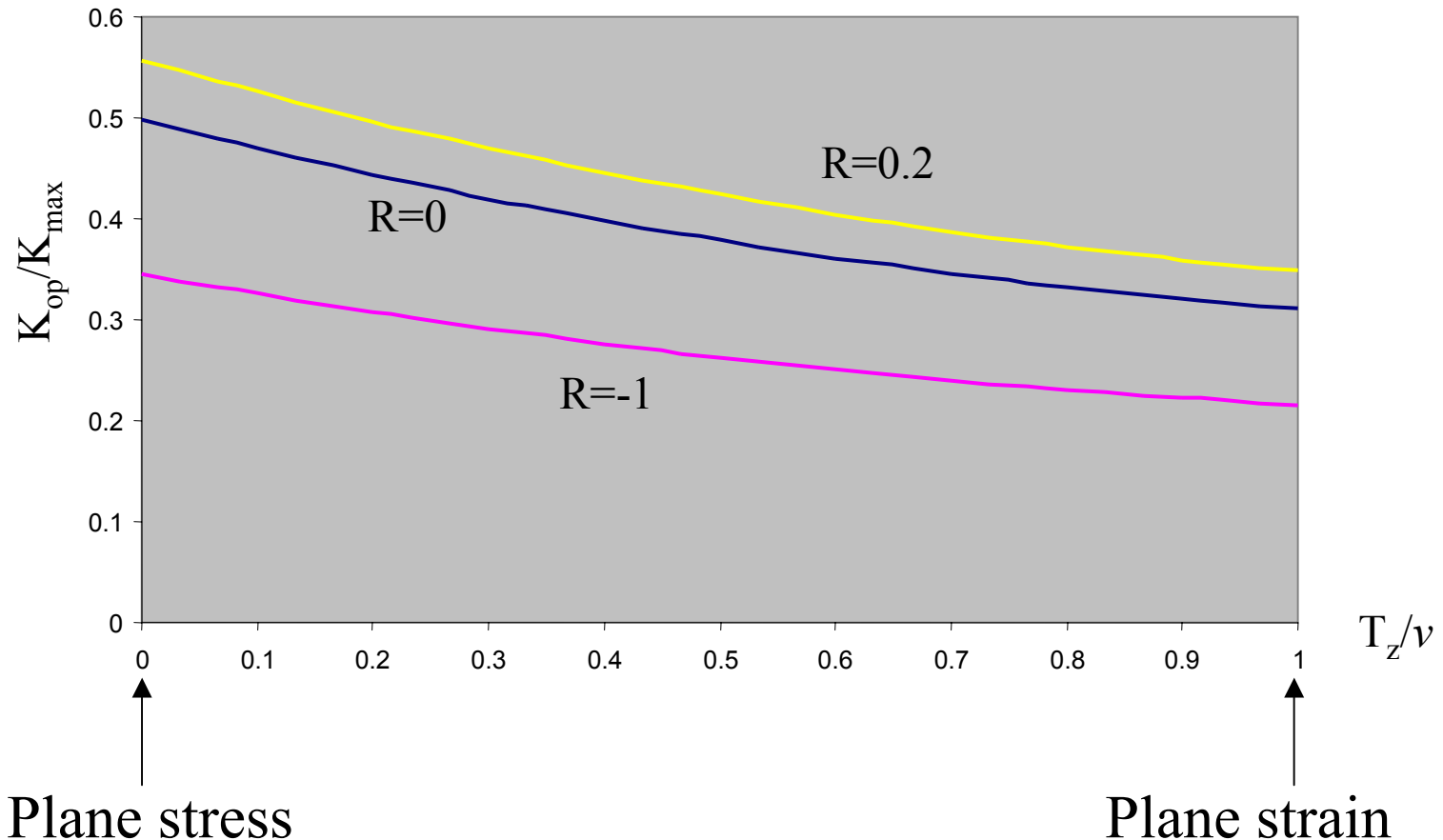
$$R = \frac{K_{min}}{K_{max}}$$

$$\Delta K_{eff} = K_{max} - K_{op}$$

$$\Delta K_{eff} = (1 - f)K_{max}$$

Plastic Zone model

2024-T3 AL

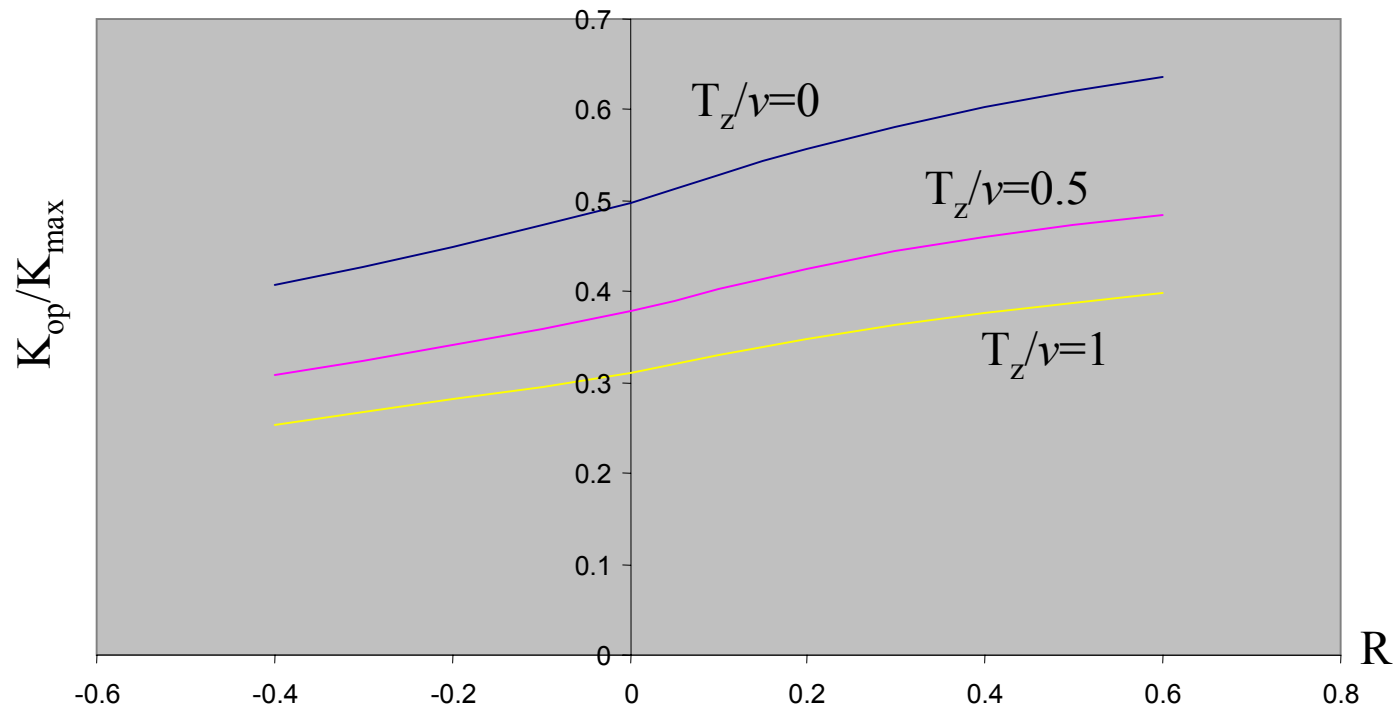


The crack propagation rate of the plane stress is less than that of the plane strain

Plastic Zone model

Plane stress

2024-T3 AL

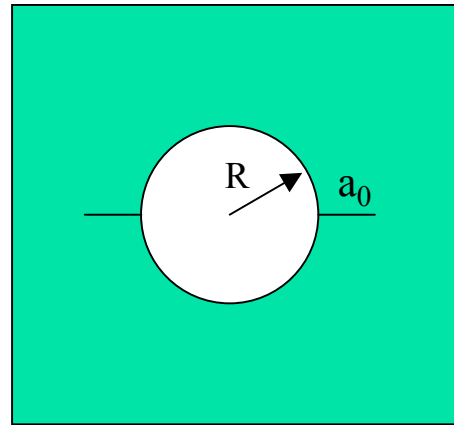


The variation of the crack opening SIF with ratio R under different 3D constraints

Plane strain

Effect of Cold-working on Fatigue

2024-T3 AL



For a single hole in a sheet, for cold working

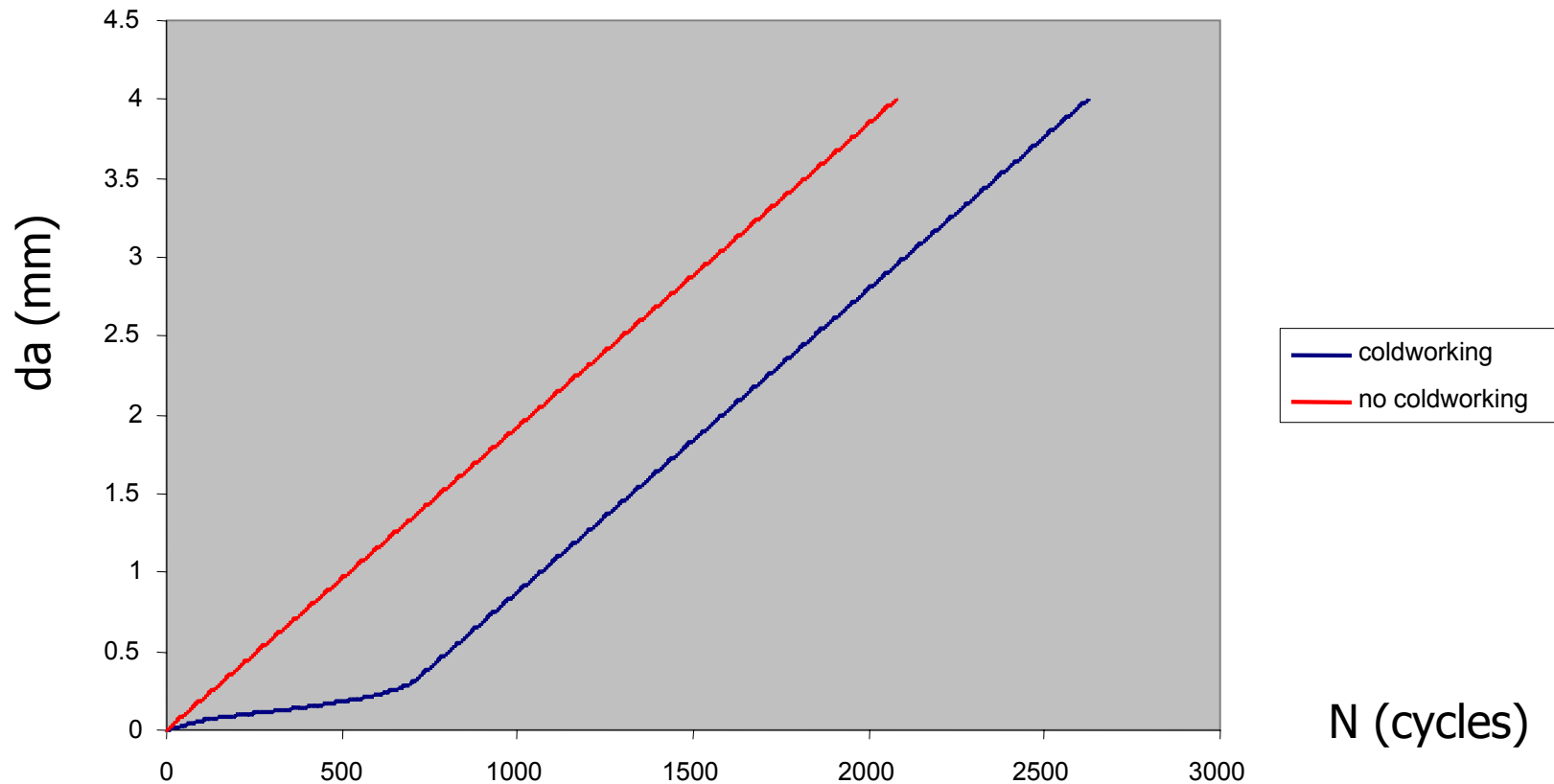
The radial pressure p_0 on the hole surface is $0.5\sigma_{ys}$

$R=2$ mm, $a_0=4$ mm, $da=4$ mm

$C=2.383E-11$

$n=3.2$

Effect of Cold-working on Fatigue



$K_{\max} = 476.09 \text{ MPa mm}^{1/2}$

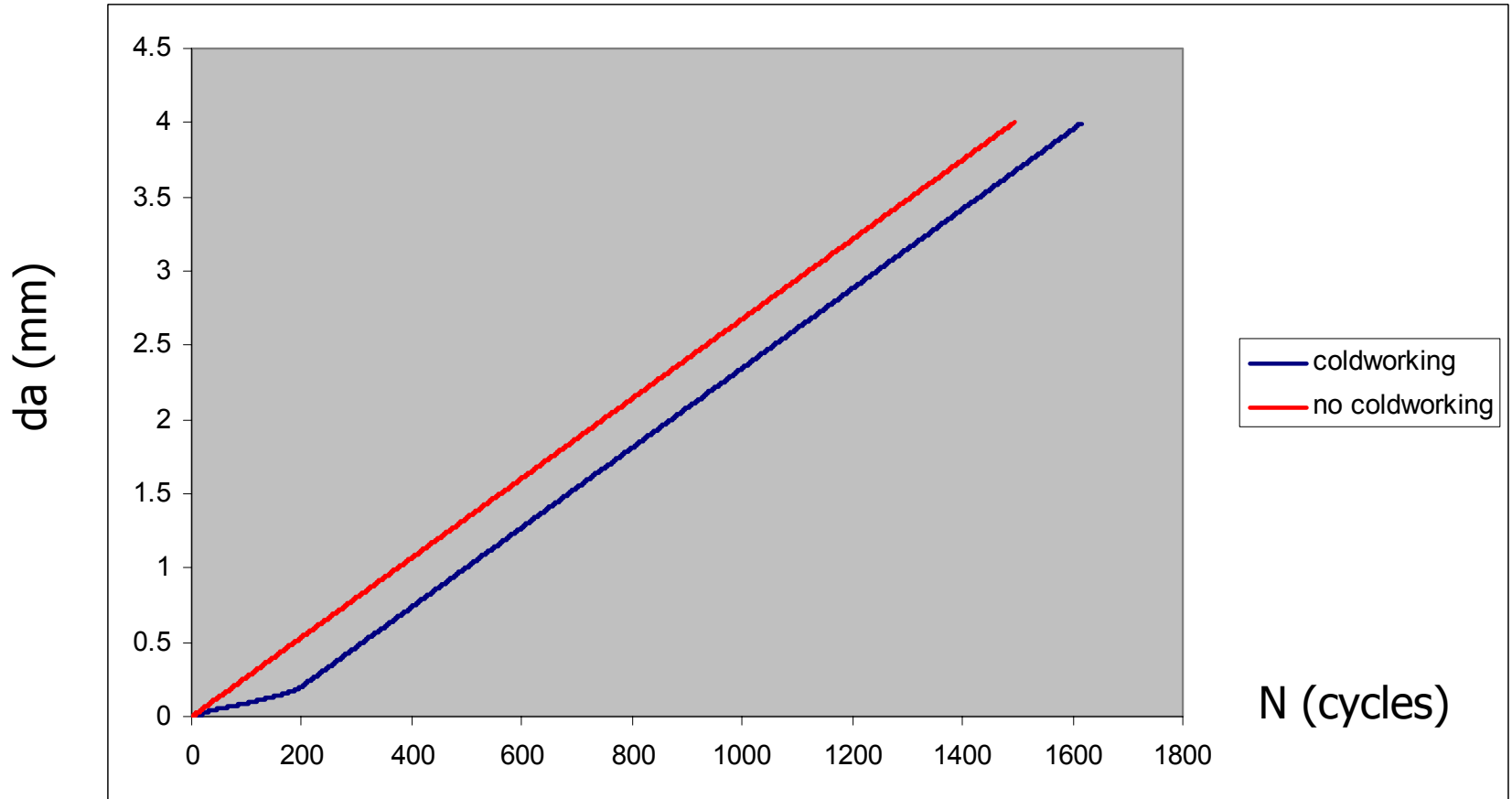
$R=0$

2,652 cycles

$T_z/v=0.5$

2,051 cycles

Effect of Cold-working on Fatigue



$K_{\max} = 476.09 \text{ MPa mm}^{1/2}$

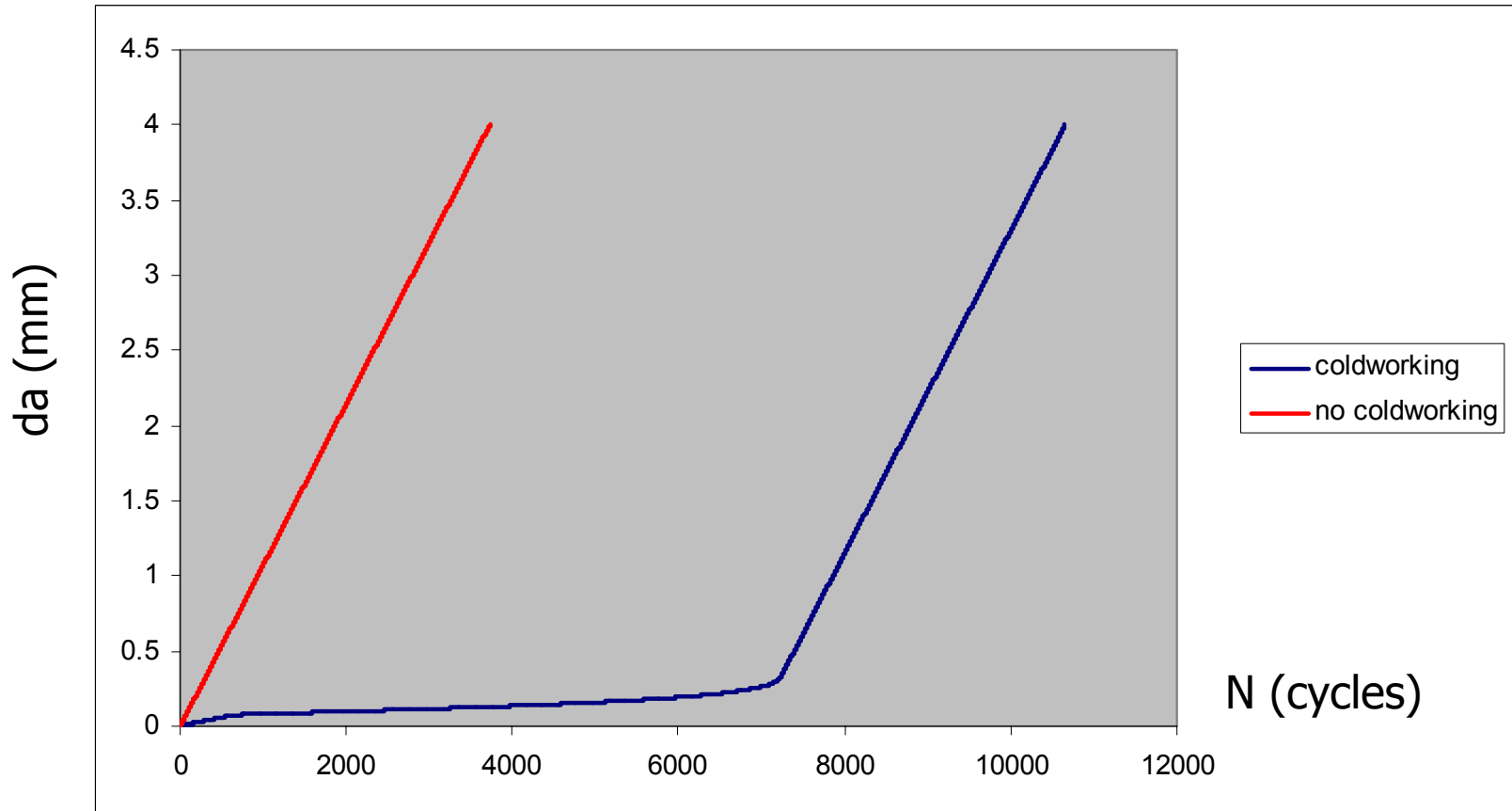
$R=0$

1,617 cycles

$Tz/v=1$

1,495 cycles

Effect of Cold-working on Fatigue



$K_{\max} = 396.75 \text{ MPa mm}^{1/2}$

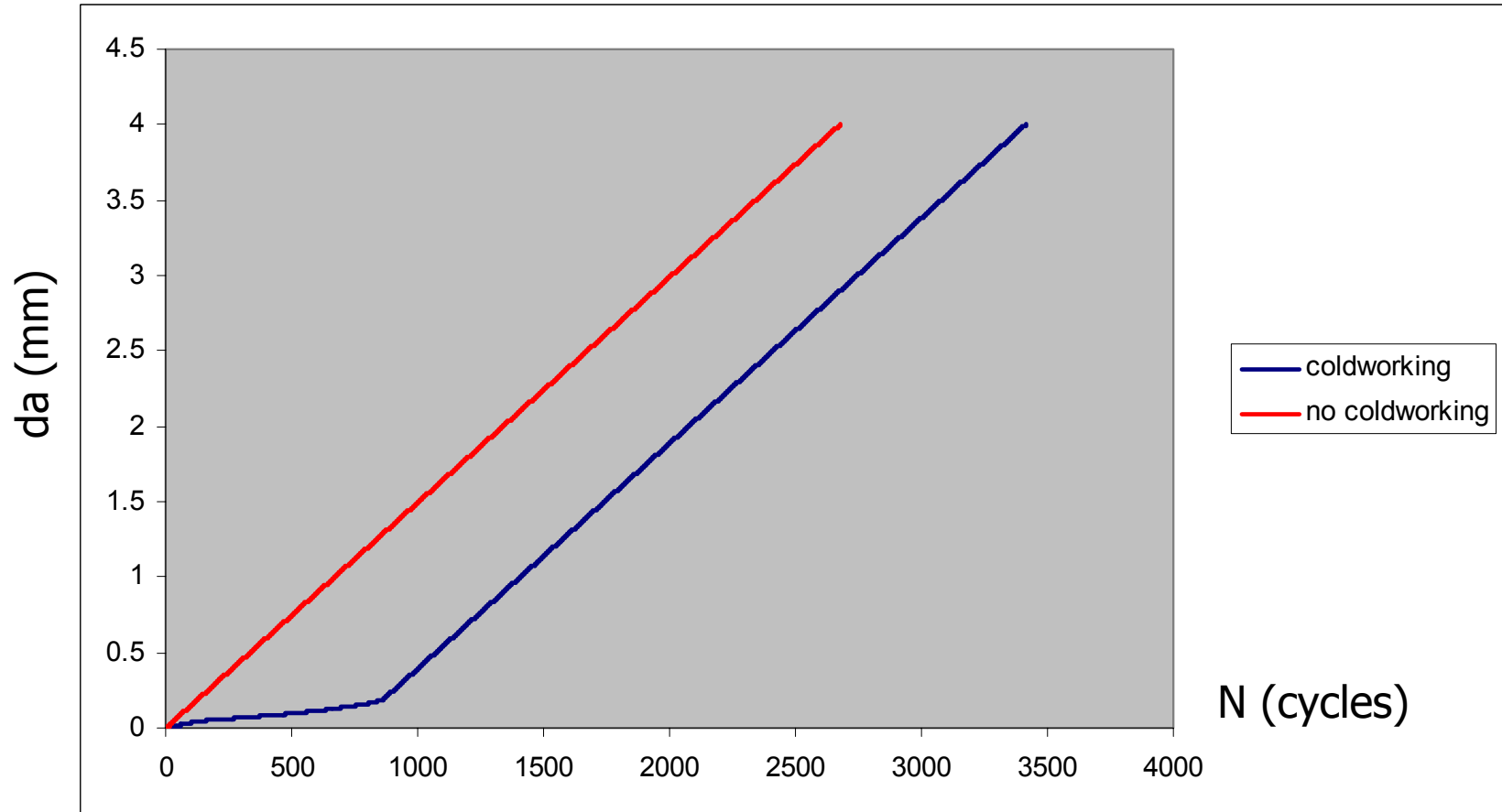
$R=0$

10,655 cycles

3,728 cycles

$T_z/v=0.5$

Effect of Cold-working on Fatigue



$K_{\max} = 396.75 \text{ MPa mm}^{1/2}$

$R=0$

3,414 cycles

$T_z/v=1$

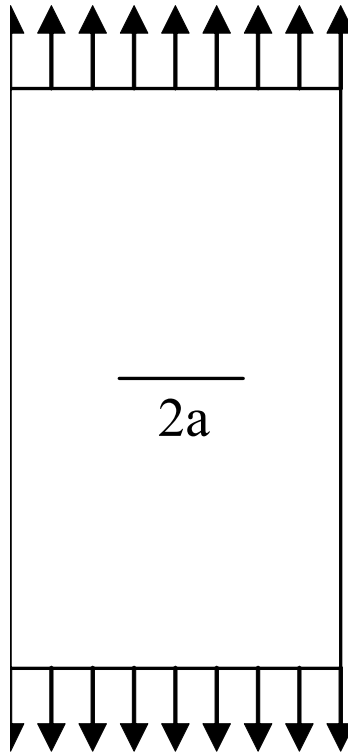
2,678 cycles

Validation of the Fatigue Model

σ_0

350WT steel

center crack



Maximum stress (σ_{\max})=114Mpa

Stress ratio $R=0.1$

$C=1.02e-8$

$n=2.94$

$a=22.4\text{mm}$

Yield strength =350 MPa

Plane stress, $T_z/v=0$

Validation of the Fatigue Model

Spectra

Constant Amplitude	Maximum stress (σ_{\max})=114Mpa Stress ratio $R=0.1$
Case 1: $R_m=1.25$	2 overloads occur at 30 and 50 mm, respectively
Case 2: $R_m=1.5$	3 overloads occur at 30, 40, and 50 mm, respectively
Case 3: $R_m=1.75$	2 overloads occur at 30 and 50 mm, respectively

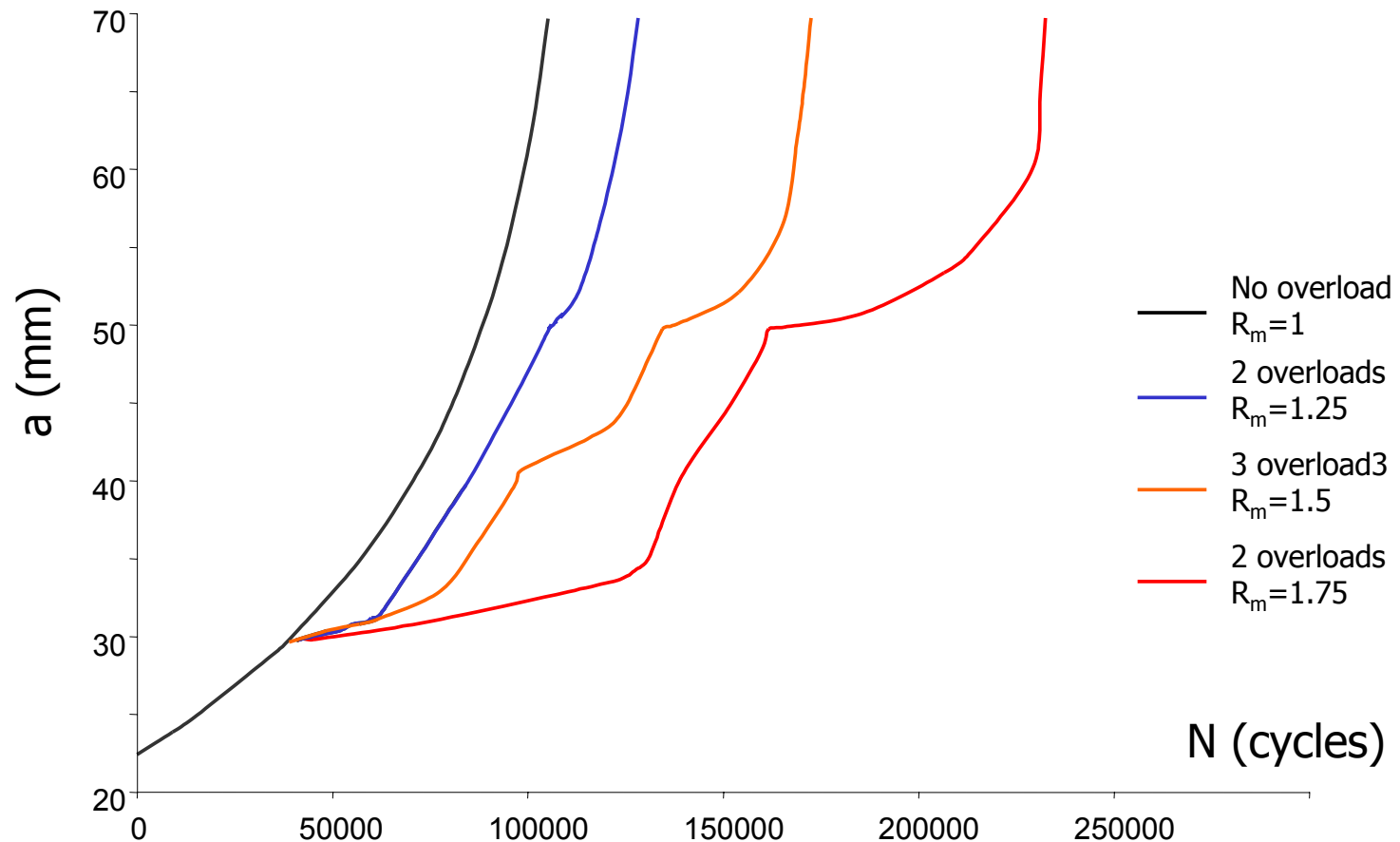
$$R_m = \frac{K_{1max}}{K_{max}}$$

Validation of the Fatigue Model

	Fatigue life: Experimental results (cycle)	Fatigue life: This model (cycle)
Case 1: $R_m=1.25$, 2 overloads	146,000	127,859
Case 2: $R_m=1.5$, 3 overloads	193,000	172,140
Case 3: $R_m=1.75$, 2 overloads	255,000	233,395

Comparing with experimental results [Taheri, et al. (2003),
Marine Structures, 16: 69-91]

Validation of the Fatigue Model

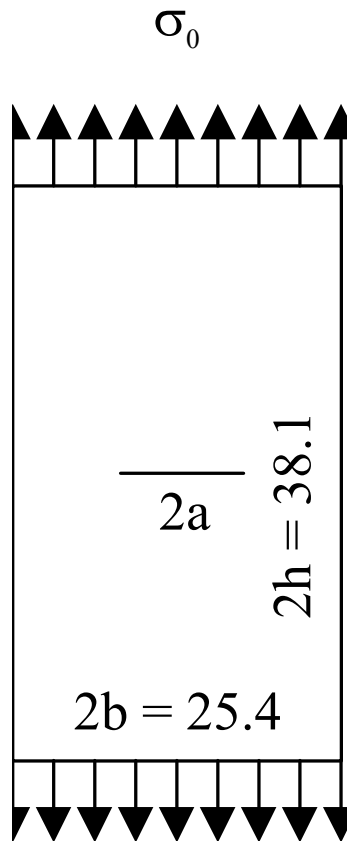


$T_z/v=0$

Numerical results

Validation of the Fatigue Model

center crack



spectrum

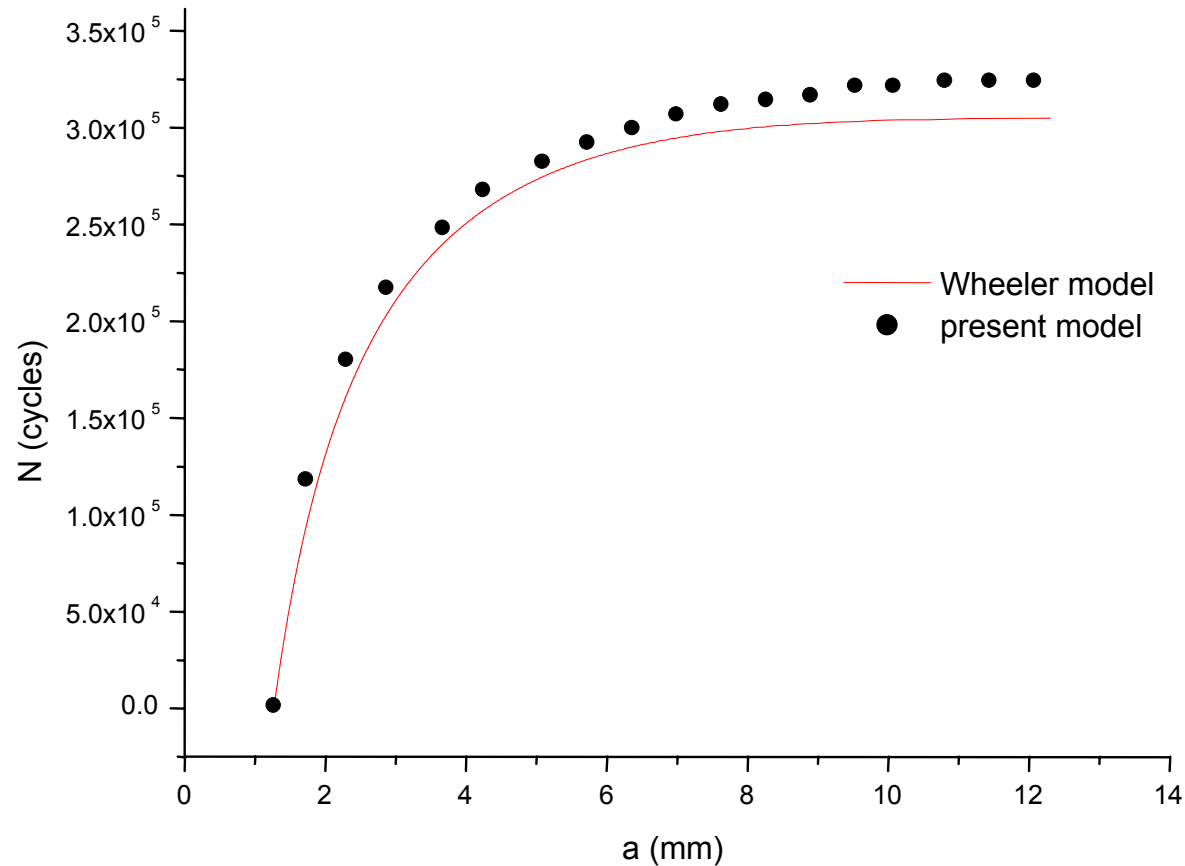
1.9MPa of 525 cycles
2.3 MPa of 255 cycles
2.44 MPa of 95 cycles
2.7 MPa of 15 cycles
3.2 MPa of 75 cycles

R=0

Fatigue model

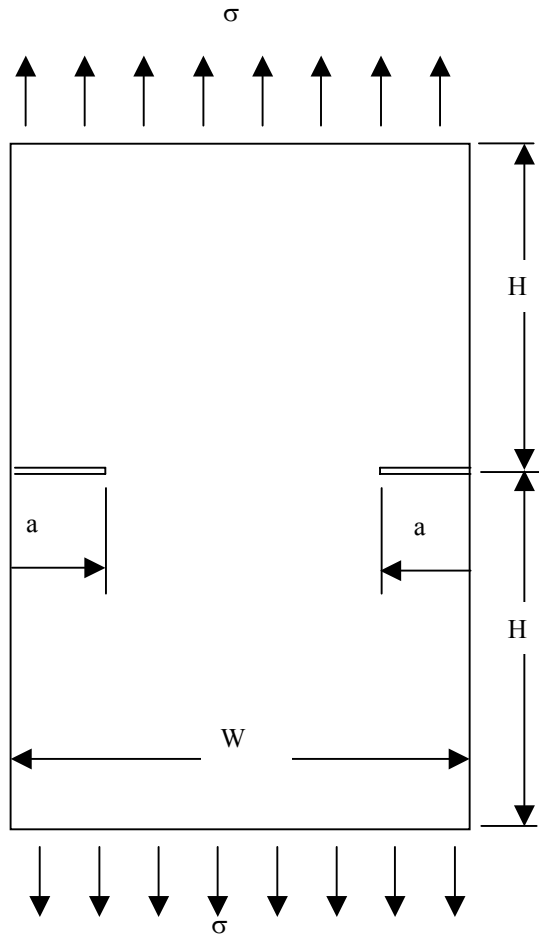
$c=1.60E-8$, $n=3.59$

Validation of the Fatigue Model



Validation of the Fatigue Model

Two edge crack



spectrum

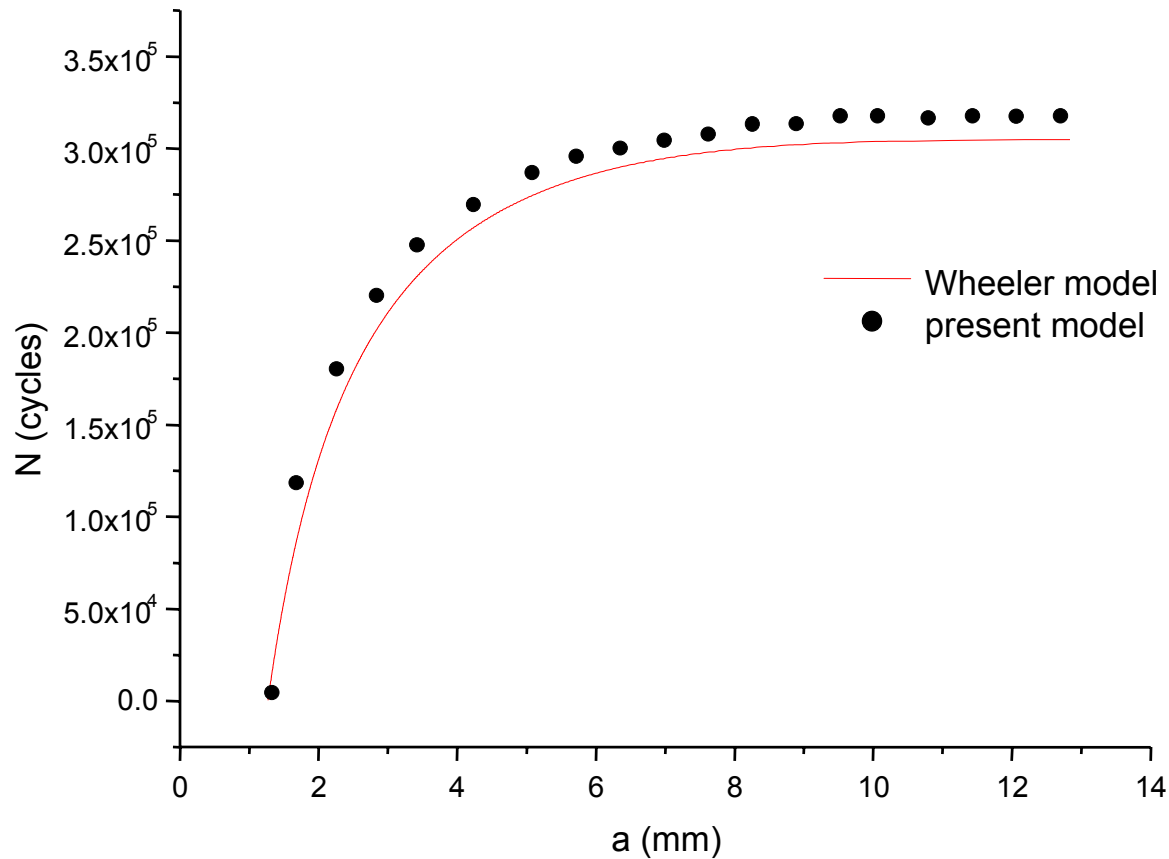
1.9psi of 525 cycles
2.3 psi of 255 cycles
2.44 psi of 95 cycles
2.7 psi of 15 cycles
3.2 psi of 75 cycles

$R=0$

Fatigue model

$c=1.49E-8$, $n=3.321$

Validation of the Fatigue Model



Conclusions

- ▶ An analytical model to model the rivet misfit is developed.
- ▶ An analytical model to model the cold-working process is developed.
- ▶ An appropriate analytical model to model the shot-peening process with 200% coverage is developed.
- ▶ The effects of residual stresses on fatigue crack growth are considered.
- ▶ A plastic zone fatigue model, which accounts for the 3D effects and the residual stress is developed to

Conclusions

- ▶ The proposed analytical model for the shot-peening process with 200% coverage can simulate the experiment well.
- ▶ The effects of residual stresses on fatigue crack growth are considered.
- ▶ The developed plastic zone fatigue model, which accounts for the 3D effects and the residual stress, is verified by the existing experiment.

Work Plan

TASK	Deliverables	Schedule
1. Completing the hardcopies and electronic version of the Detailed Work Plan.	Detailed Work Plan	Month 1
2. Performing literature search for crack growth models and data for crack in residual stress fields. Identify, examine, and quantify all of the material properties, design parameters, operating conditions, and other factors that impact fatigue life in rotorcraft.	Technical Report on Material Properties, Design Parameters, Operating Conditions & other factor impacting fatigue life in rotorcraft	Month 2-3
3. Developing an appropriate analytical model to model the cold-working process.	Technical Report on analytical cold-working model	Month 4-5
4. Developing an appropriate analytical model to model the shot-peening process with 200% coverage.	Technical Report on analytical shot-peening model	Month 6

Work Plan

5. Developing the plastic strip and plastic zone models appropriate for all possible rotorcraft components and operating conditions, e.g., load interaction, environment, residual stresses caused by shot peening, rivet misfit, and cold expanded holes.	Theoretical Report on the advanced fatigue model	Month 7-8
6. Implementing Task 3-5 numerically by FEAM.	Theoretical Report	Month 9-12
7. Getting test data for cold-working process from AACE and Sikorsky, for shot-peening from Sikorsky.	Theoretical Report	Month 13-14
8. Comparing the results of fatigue crack growth of cold-working parts with those from AACE and Sikorsky.	Theoretical Report & database	Month 15-16

Work Plan

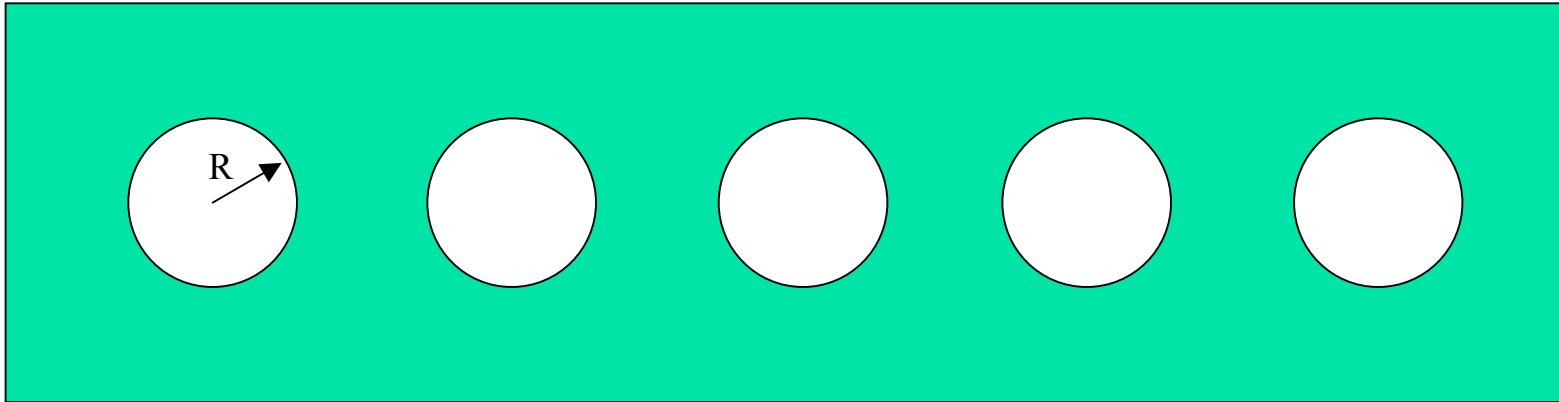
9. Comparing the results of residual stresses caused by shot-peening with those from Sikorsky.	Theoretical Report & database	Month 17-18
10. Comparing the results of fatigue crack growth of shot-peened parts with those from Sikorsky.	Theoretical Report & database	Month 19-20
11. Comparing the results of fatigue crack growth of surface crack in shot-peened parts, from the developed fatigue models in this project, with the test data provided by FAA.	Technical Report & database	Month 21-22
12. Documentation: Completing the hardcopies and electronic versions of user manuals for all analysis tools enhanced or developed; Completing the technical report summarizing the work performed and results of the analysis.	User manuals & Final Technical Report	Month 23-24

Fatigue Crack Growth for Through-thickness Crack

two examples, by considering the variation of the size of the plastic zone along the thickness of the specimen.

On the surface of the specimen, it is plane-stress status, which has large plastic zone. While in the center of the specimen, it is plane-strain, which has small plastic zone, about $1/3$ of the size of the plastic zone on the surface. Thus, the crack open stress on the surface is greater than that in the center. Hence, the crack growth rate on the surface is less than that in the center.

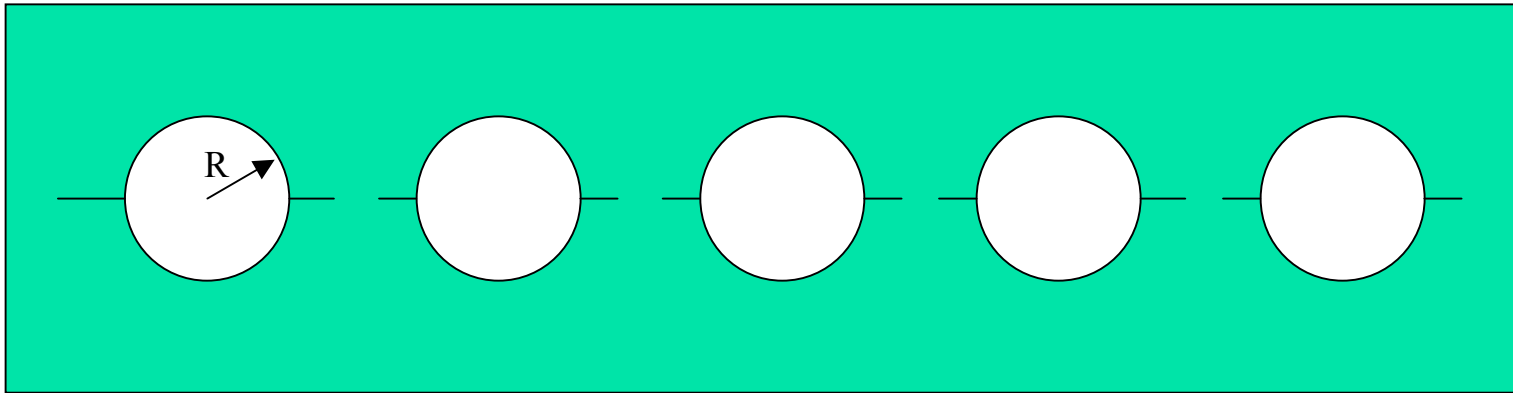
Effect of Residual Stresses in the Fastener Hole



Assume that the rivet misfit is equal to v_0 for all the fastener holes in a row. Without any far field loading, the initial radial pressure p_0 on each hole is also

$$p_0 = (2\mu) \frac{v_0}{R} = k_0 \frac{v_0}{R}$$

Effect of Residual Stresses in the Fastener Hole



When cracks are present near the holes, the initial radial pressure on each hole will be a function of the crack length

$$p_i = k_i \frac{v_0}{R} \quad \text{or} \quad k_i = \frac{Rp_i}{v_0}$$

The initial radial pressure p_i is solved for, using the FEAM.

The stiffness k_i depends on the lengths of the cracks emanating from the i th hole.

Effect of Residual Stresses in the Fastener Hole

Let the applied far-field stress be σ_1 , and the maximum v displacement at the i th hole due to σ_1 alone be designated as v_{i1} . v_{i1} is determined from the FEAM, and is a function of the lengths of the cracks emanating from the i th hole.

The radial pressure exerted due to initial fastener-misfit, is given during the course of far-field loading, by

$$p_{i1} = \begin{cases} k_i |v_{i1} - v_0|/R & \text{when } v_{i1} < v_{i0} \\ 0 & \text{when } v_{i1} \geq v_{i0} \end{cases}$$

Effect of Residual Stresses in the Fastener Hole

A far-field zero-to tension cyclic load: 0 to σ_1 at the upper edge;
and 0 to σ_0 at the lower edge

The maximum SIF at the crack at the i th hole is

$$K_{i\max} = K_i + K_{i1}$$

The minimum SIF at the crack at the i th hole is $K_{i\min} = K_{i0}$

K_i : the SIF due to far-field alone.

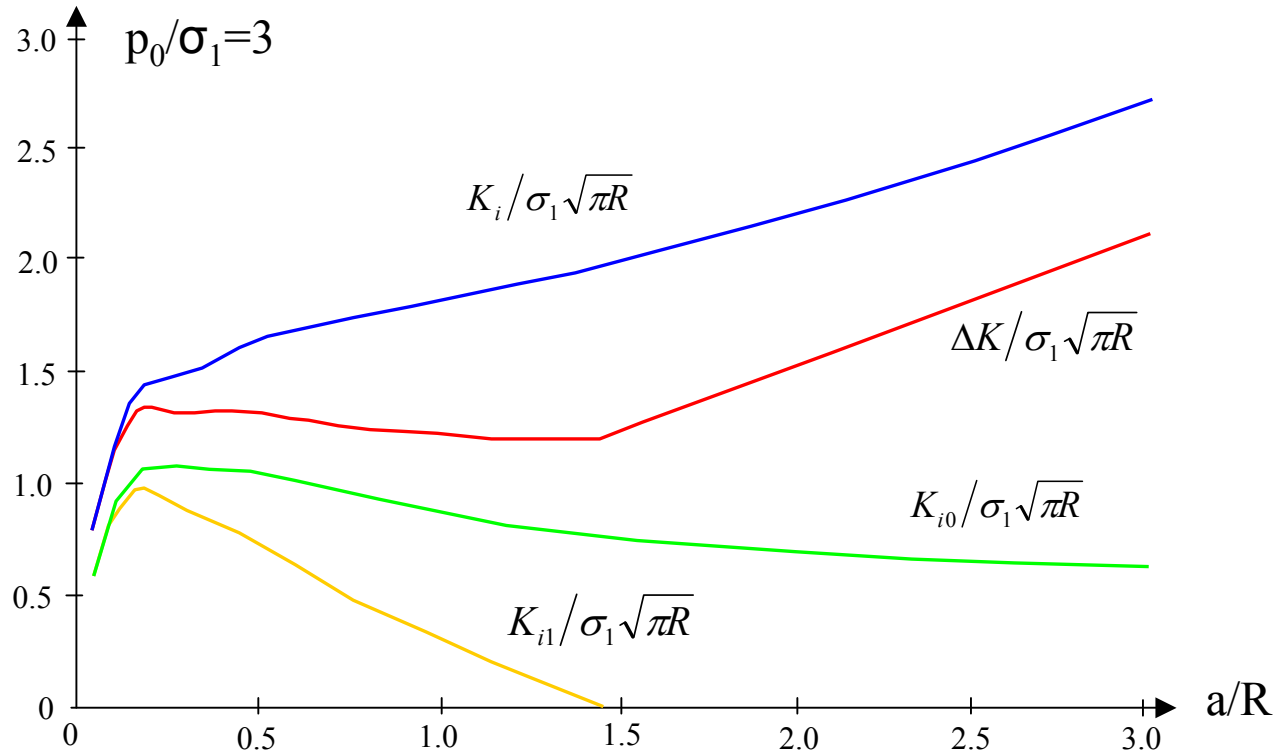
K_{i0} : the SIF due to the initial radial (at zero far-field tension)
pressure due to fastener misfit.

K_{i1} : the SIF due to the residual pressure p_{i1} when applying far-field stress.

The residual stresses affect fatigue crack growth by two factors:

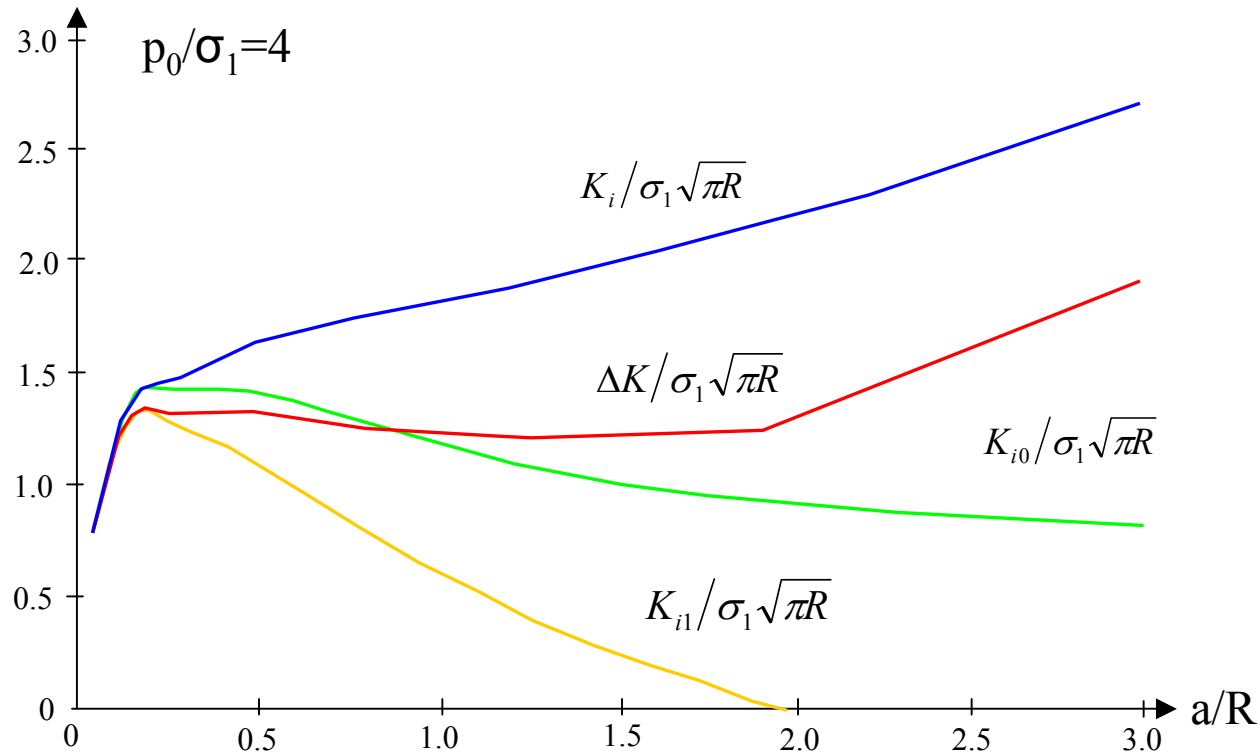
- Reducing the SIF rang ΔK ;
- Increasing the stress-ratio.

Effect of Residual Stresses in the Fastener Hole



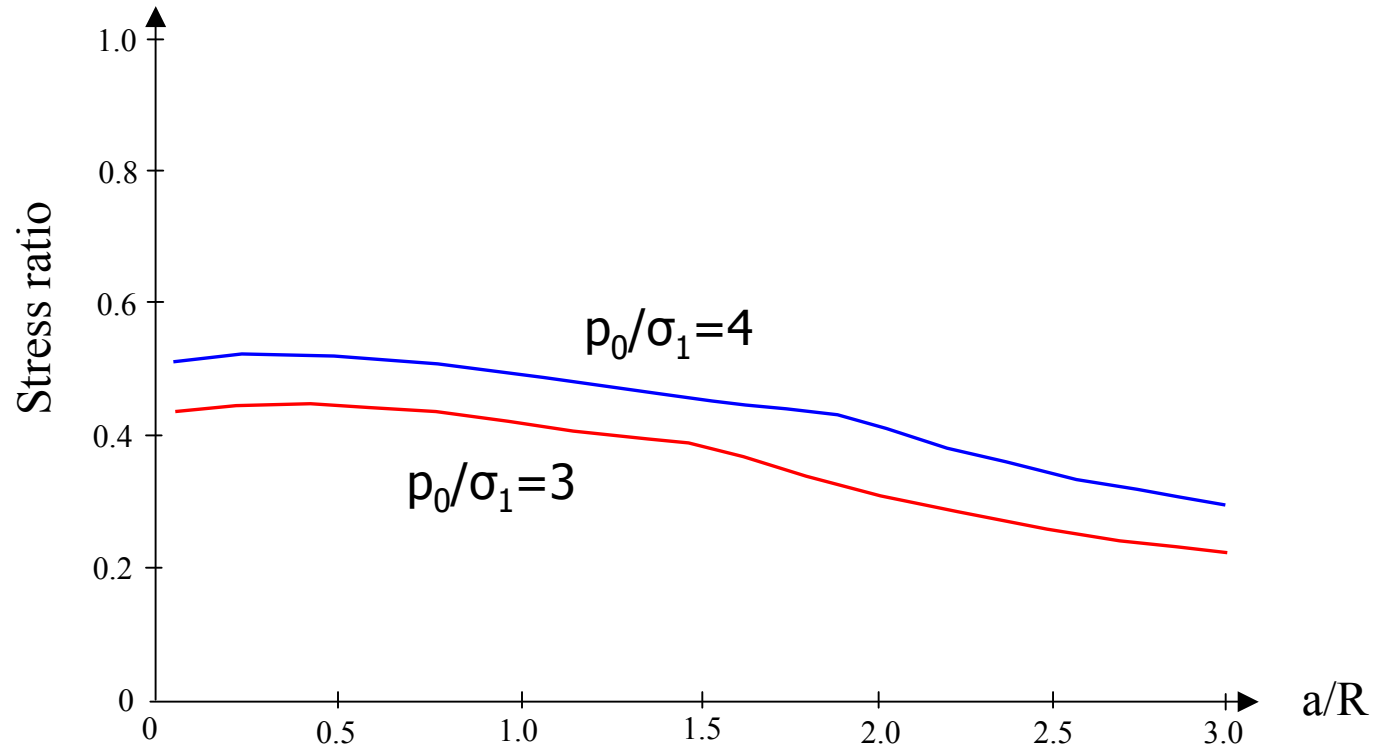
Variation of SIF and SIF rang as functions of (a/R) , with the effect of residual stresses being considered ($p_0/\sigma_1=3$)

Effect of Residual Stresses in the Fastener Hole



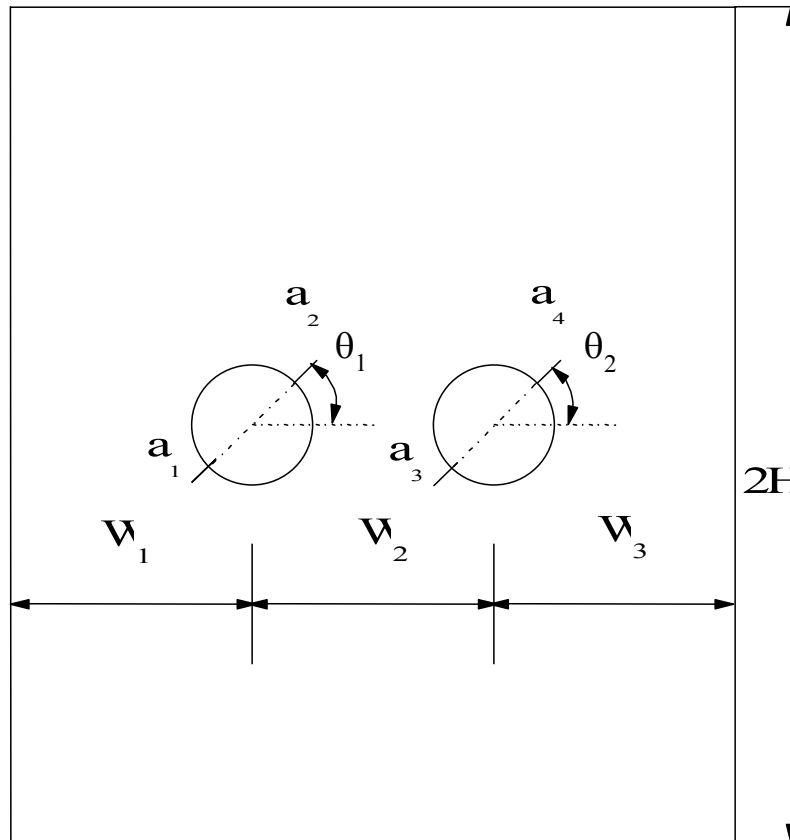
Variation of SIF and SIF rang as functions of (a/R) , with the effect of residual stresses being considered ($p_0/\sigma_1=4$)

Effect of Residual Stresses in the Fastener Hole



Variation of stress ratio as a function of a/R

Fatigue crack growth behavior of cracks emanating from fastener holes

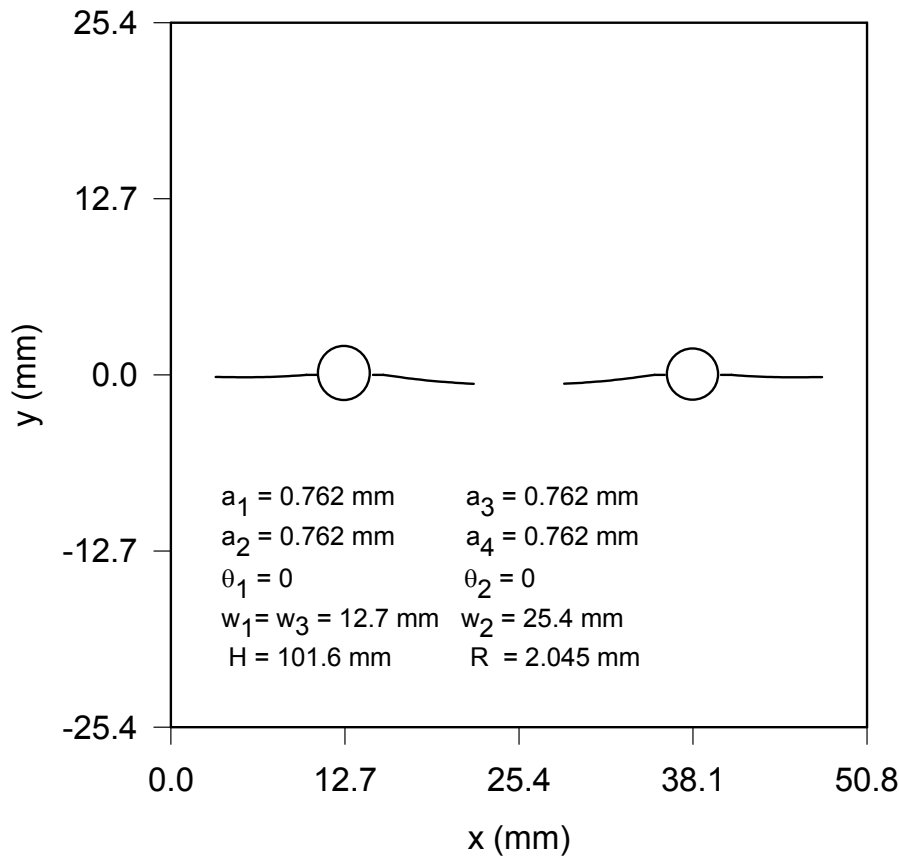


2024-T3 Aluminium alloy

The fastener-load is distributed along the periphery of the hole, by using the analytical solution for the contact problem between the rivet and the hole. In this example, for simplicity, the fastener load is distributed sinusoidally.

The initial crack configuration

Fatigue crack growth behavior of cracks emanating from fastener holes



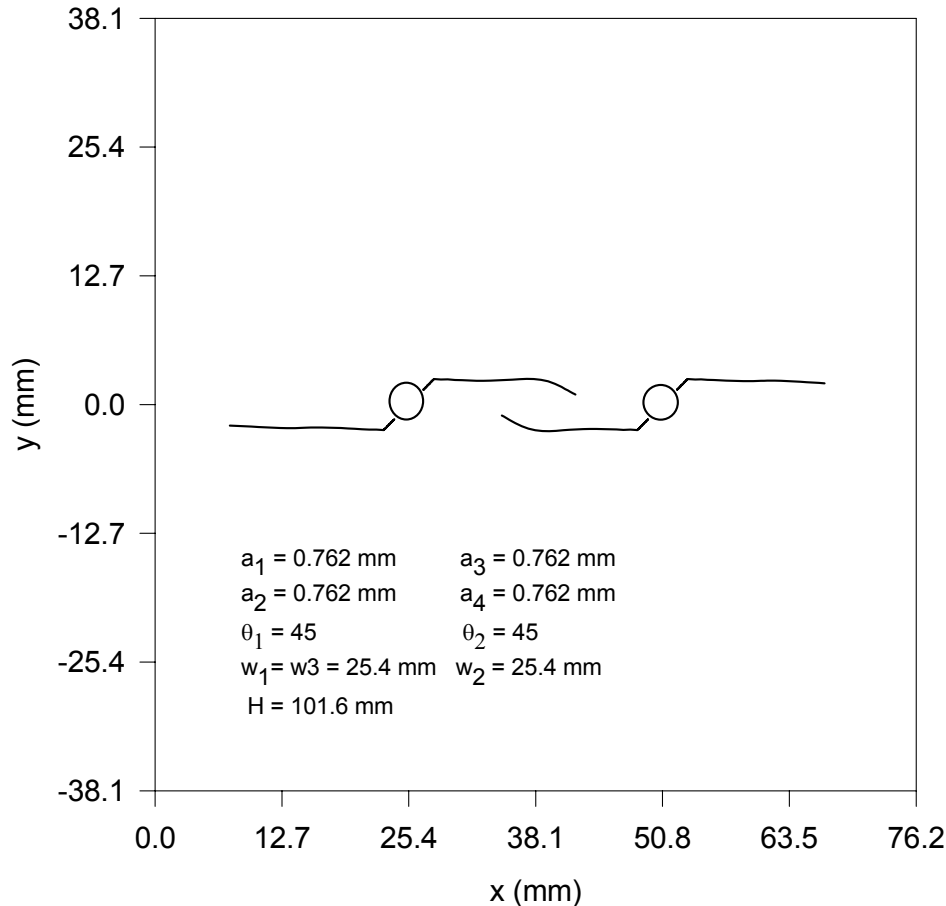
uniform stress σ_0 is applied on the upper horizontal edge, and an equilibrating sinusoidally distributed pin loading exists on the lower half of the hole periphery.

Stress $\sigma_0 = 82.74$ MPa

Stress ratio = 0.1

The total applied loading cycles
19,800 cycles

Fatigue crack growth behavior of cracks emanating from fastener holes



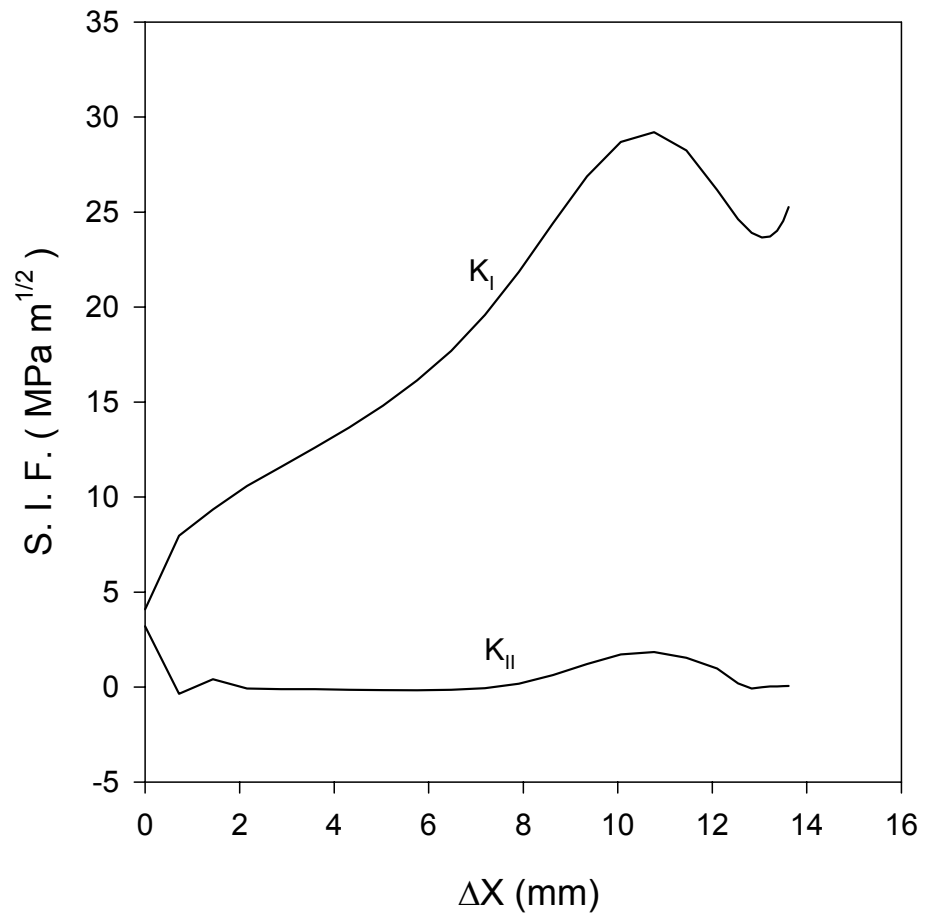
The loading consists of uniform stress σ_0 on the upper and lower edges of the sheet

Both the initial cracks emanating from the fastener holes are slanted at 45° degrees

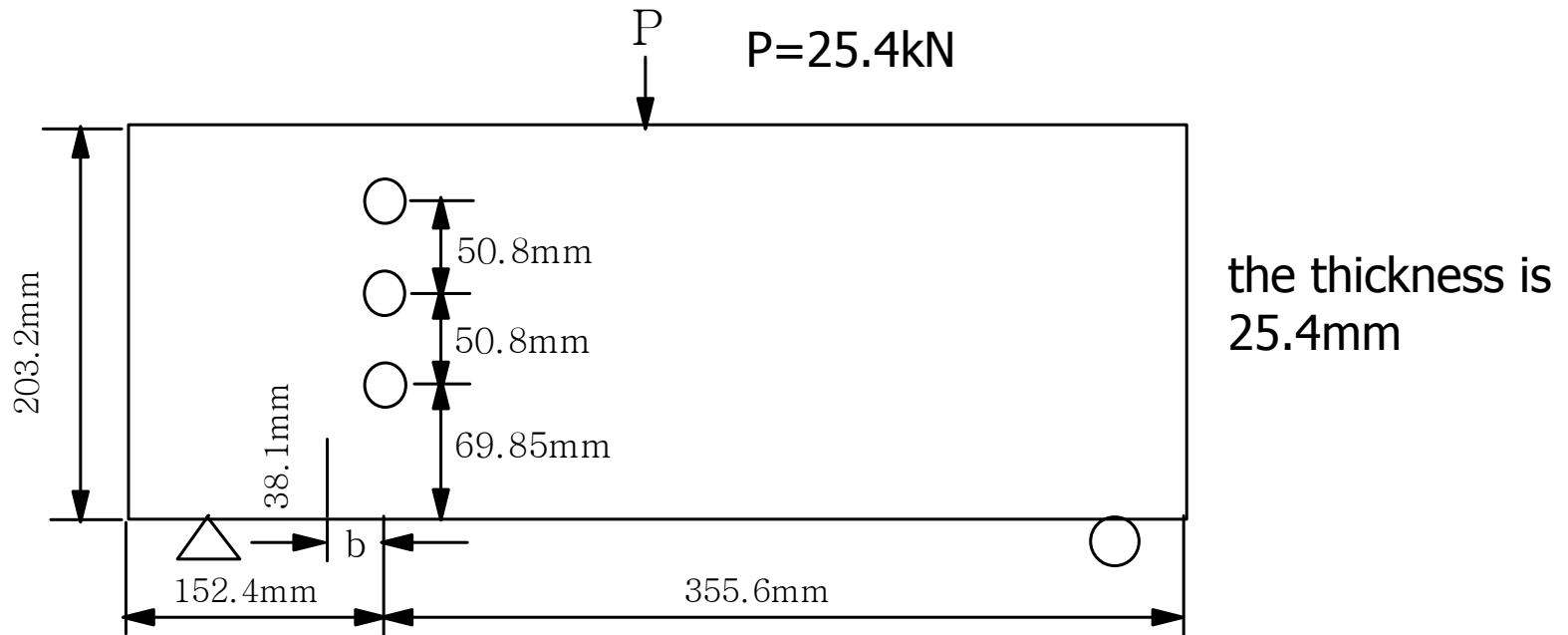
Crack growth direction is determined by using the maximum principal stress criterion

Fatigue crack growth behavior of cracks emanating from fastener holes

The variation of mode I and mode II stress intensity factors as the second crack is growing from its initial crack length a_2



Fatigue crack growth of an edge crack embedded in a beam with three rivet holes



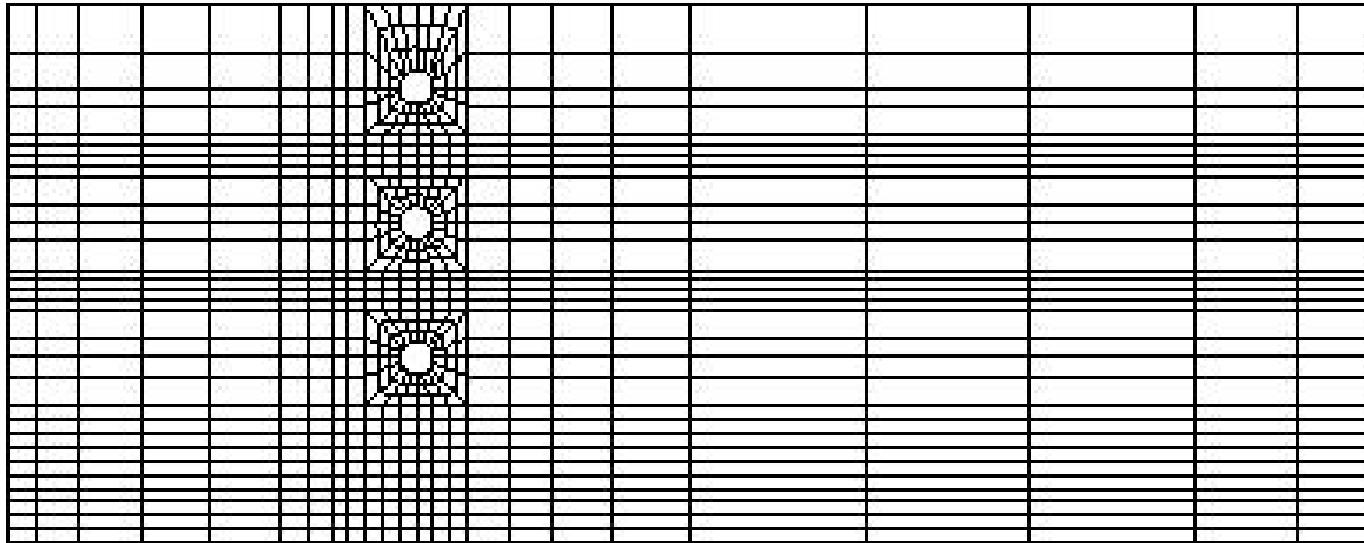
Schematic of cracked beam with rivet holes

PMMA

$E = 2.76\text{GPa}$

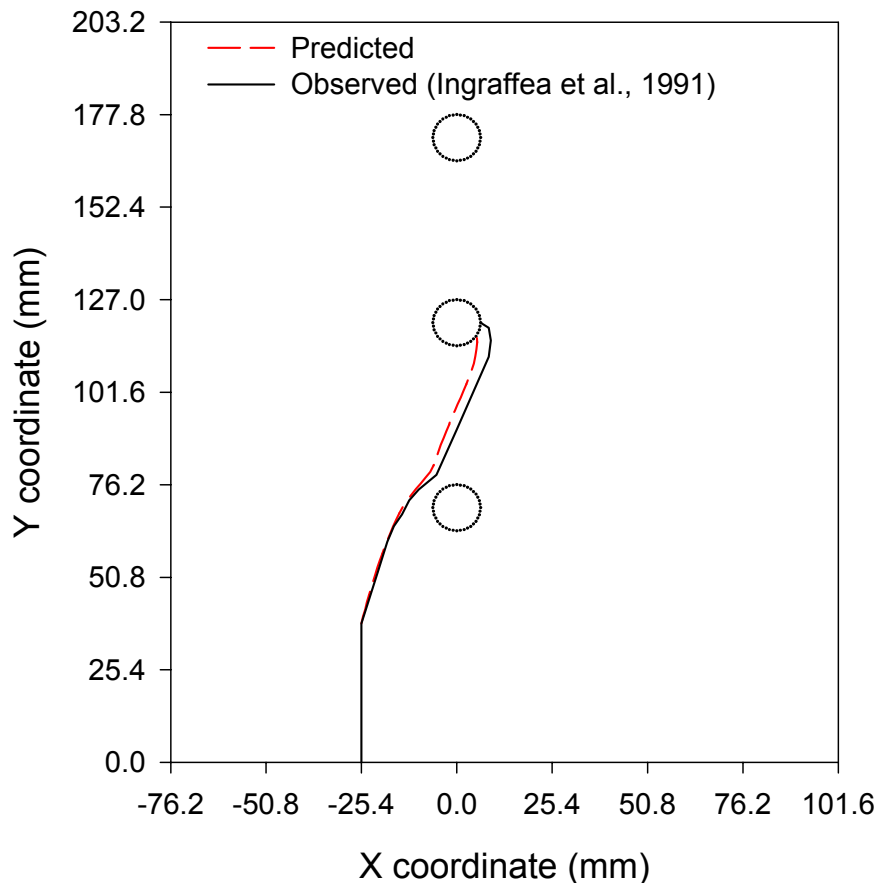
Poisson's ratio is 0.38

Fatigue crack growth of an edge crack embedded in a beam with three rivet holes



Finite element mesh for beam with rivet holes

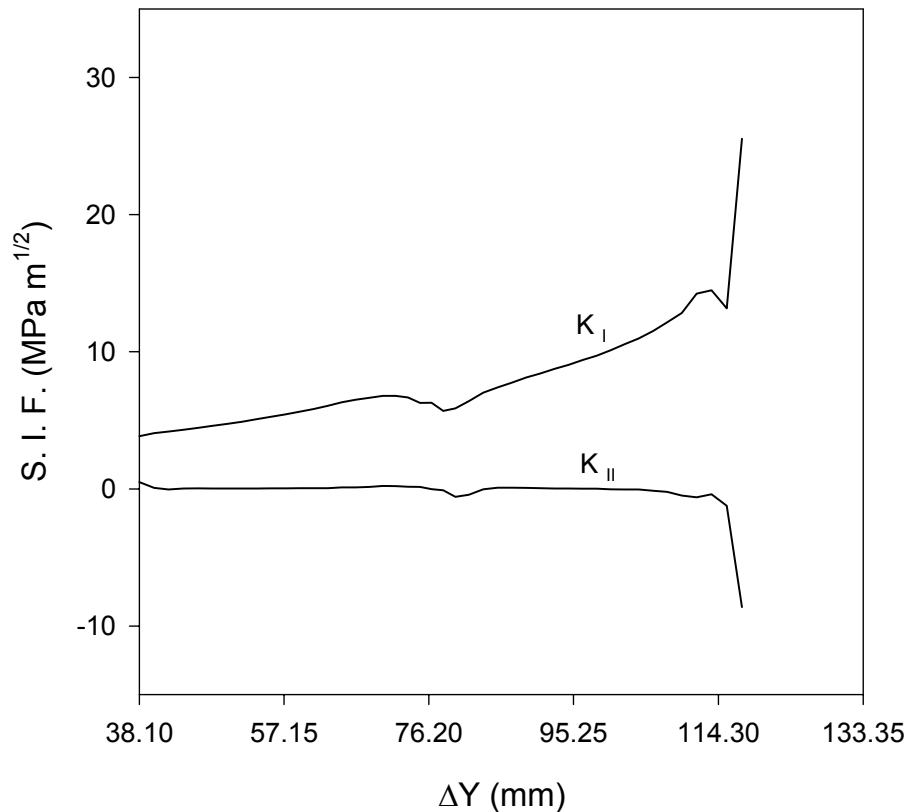
Fatigue crack growth of an edge crack embedded in a beam with three rivet holes



Simulated and experimental crack growth trajectories when $b=25.4\text{mm}$

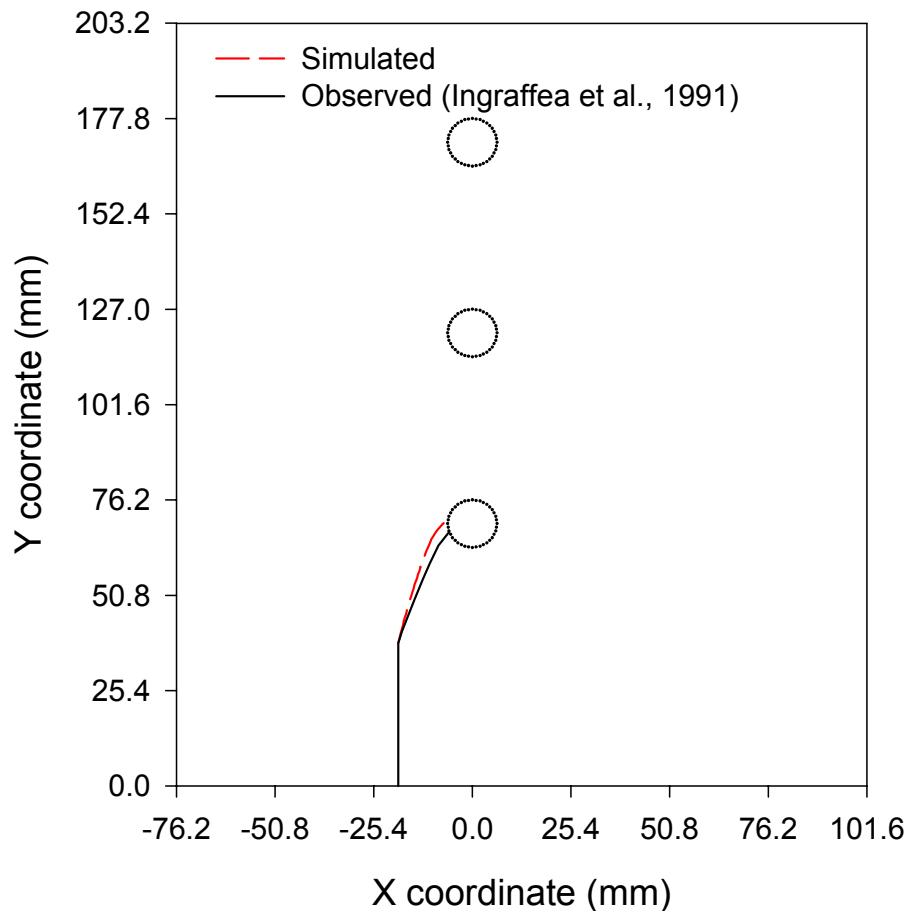
Crack growth direction is determined by using the maximum principal stress criterion

Fatigue crack growth of an edge crack embedded in a beam with three rivet holes



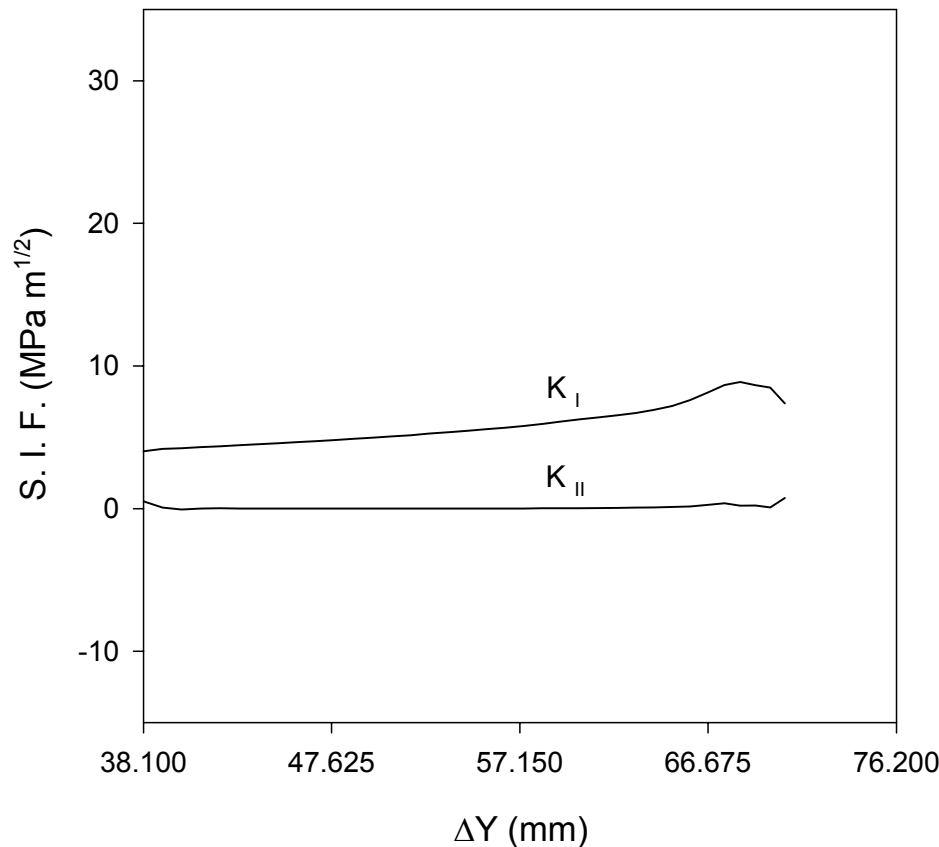
Variation of mode I and mode II stress intensity factors according to the increment of y coordinate of the crack tip when $b=25.4\text{mm}$

Fatigue crack growth of an edge crack embedded in a beam with three rivet holes



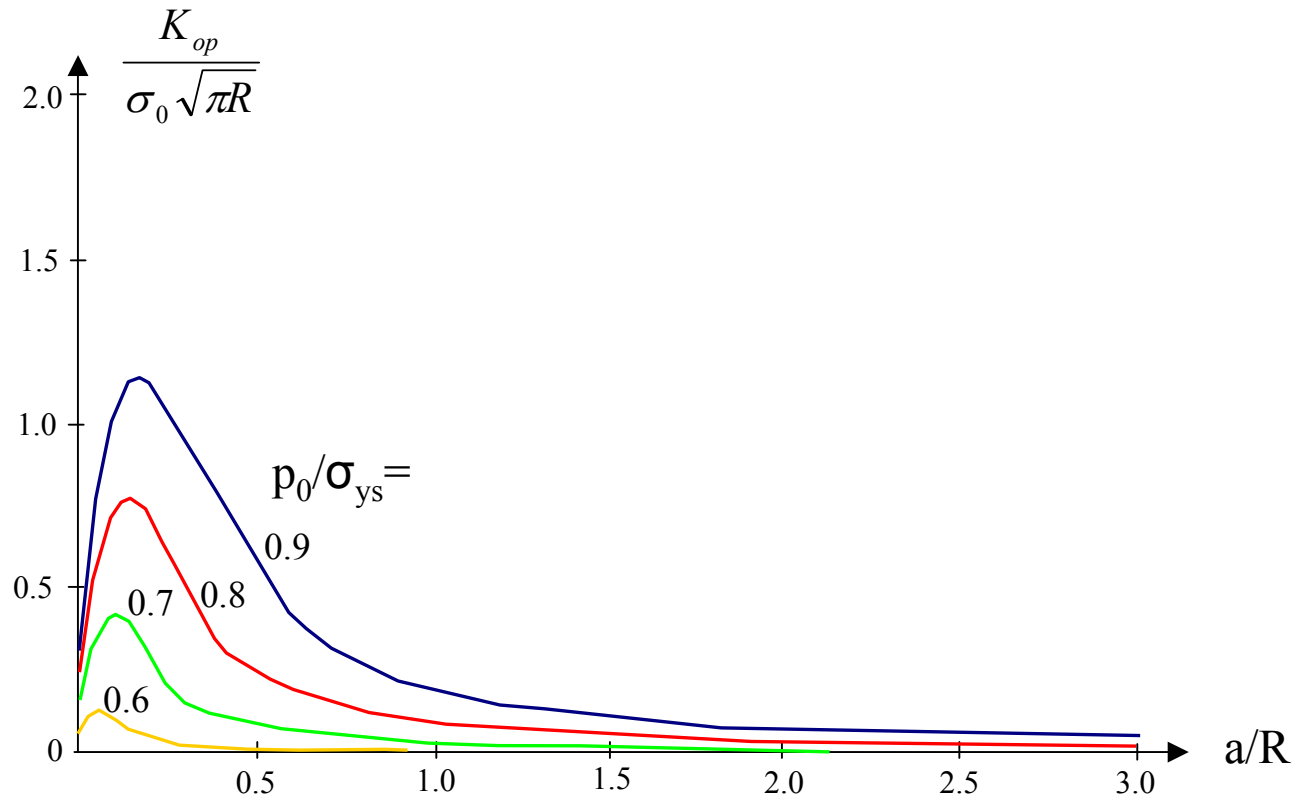
Simulated and experimental crack growth trajectories when $b=19.05\text{mm}$

Fatigue crack growth of an edge crack embedded in a beam with three rivet holes



Variation of mode I and mode II stress intensity factors according to the increment in y coordinate of the crack tip when $b=19.05\text{mm}$

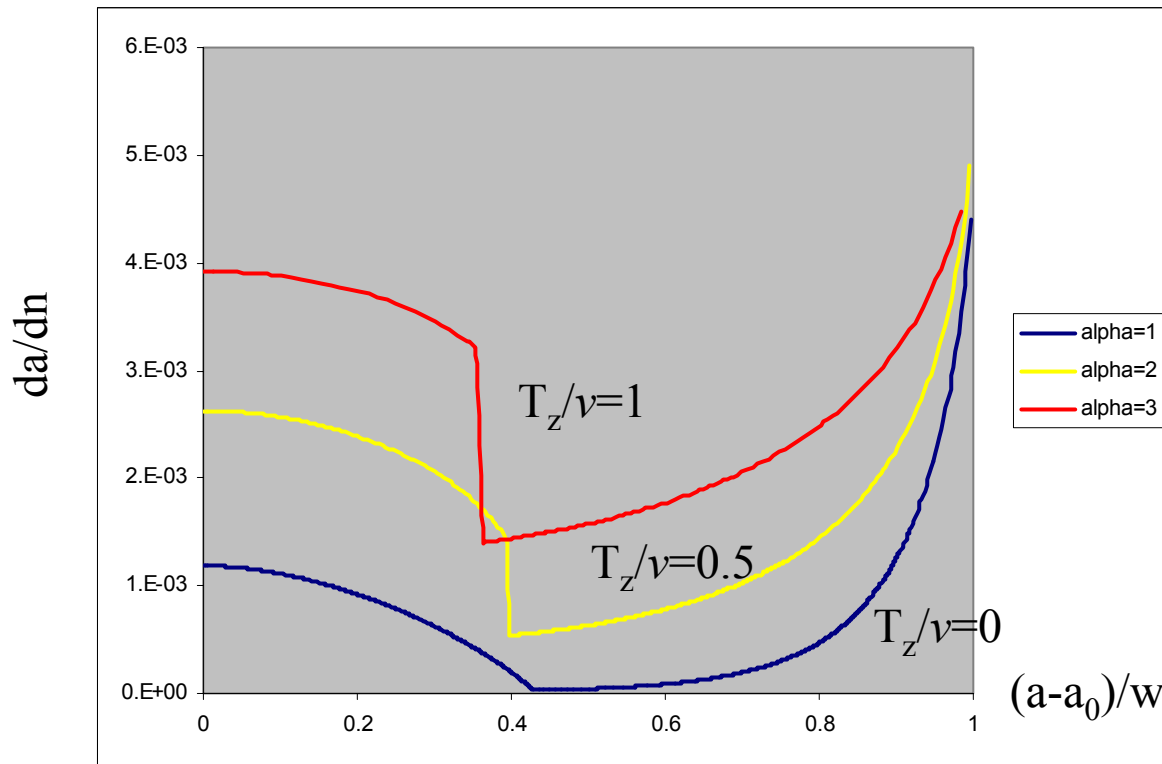
Effect of plastic deformation due to cold-working



Variation of the open SIF K_{op} for cracks of various length as compared to the radius of the plastic zone due to cold-working

Plastic Zone model

2024-T3 AL



$$K_{1\max} = 869.63 \text{ MPa mm}^{1/2}$$

$$R_m = 1.5$$

$$R_1 = -0.5$$

$$R = 0$$

$$C = 2.383 \times 10^{-11}$$

$$n = 3.2$$

5,219 cycles

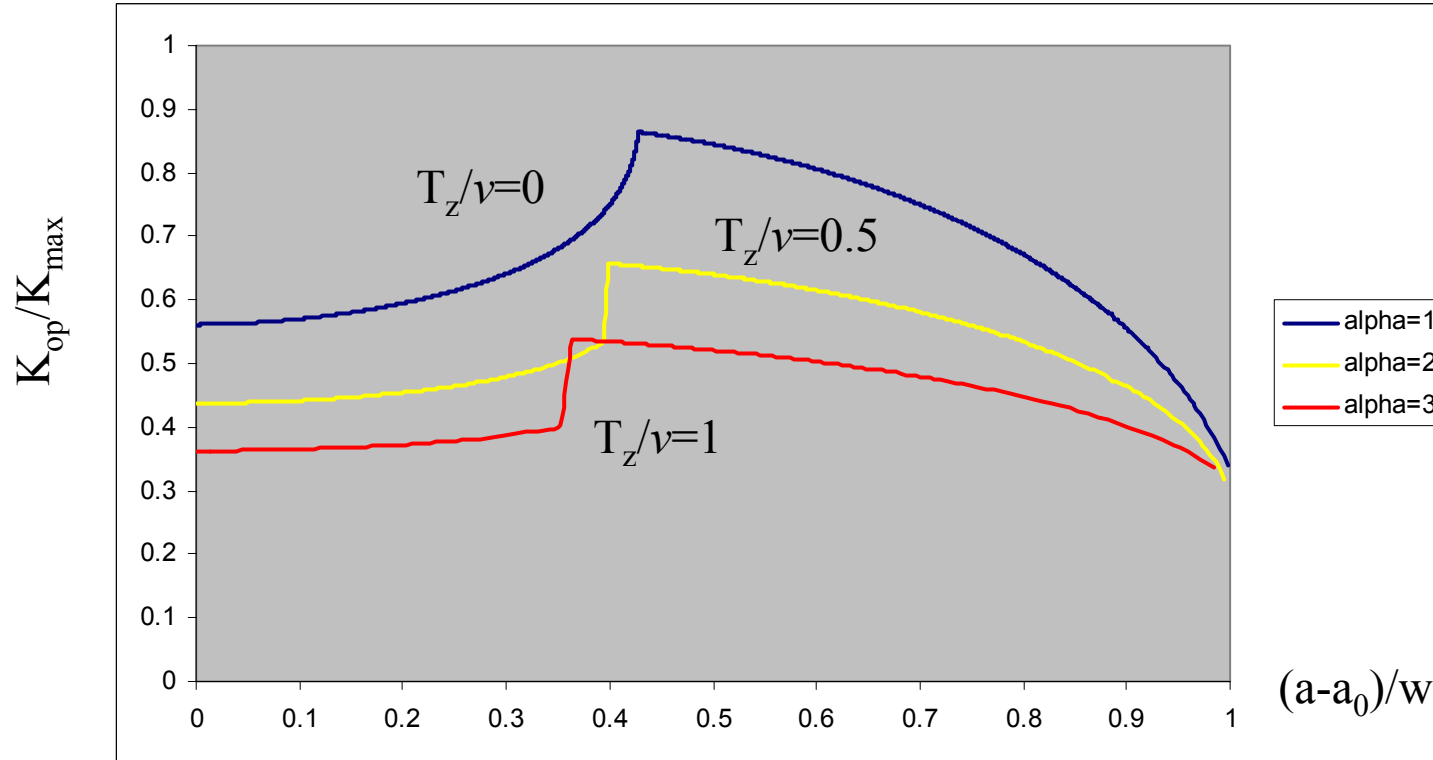
383 cycles

126 cycles

The crack propagation rate, following an overload, under different 3D constraints

Plastic Zone model

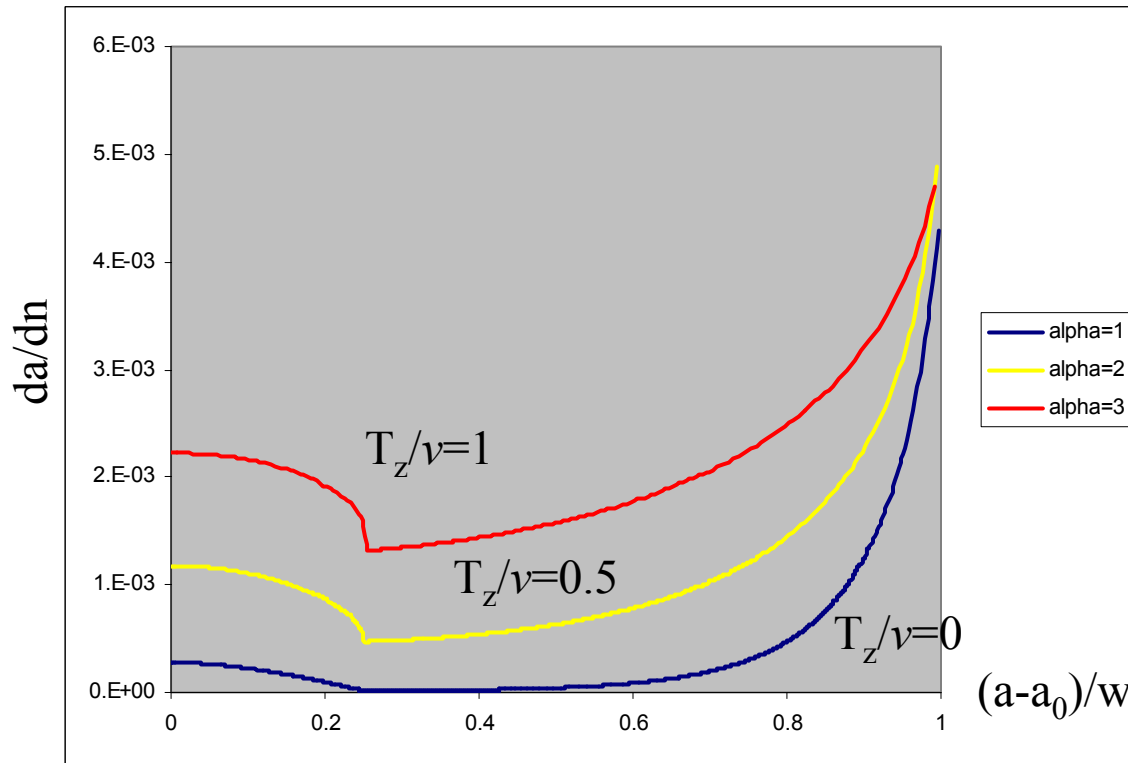
2024-T3 AL



The crack opening SIF following an overload, under different 3D constraints

Plastic Zone model

2024-T3 AL



$$K_{I\max}=869.63 \text{ MPa mm}^{1/2}$$

$$R_m=1.5$$

$$R_1=0$$

$$R=0$$

$$C=2.383E-11$$

$$n=3.2$$

14,185 cycles

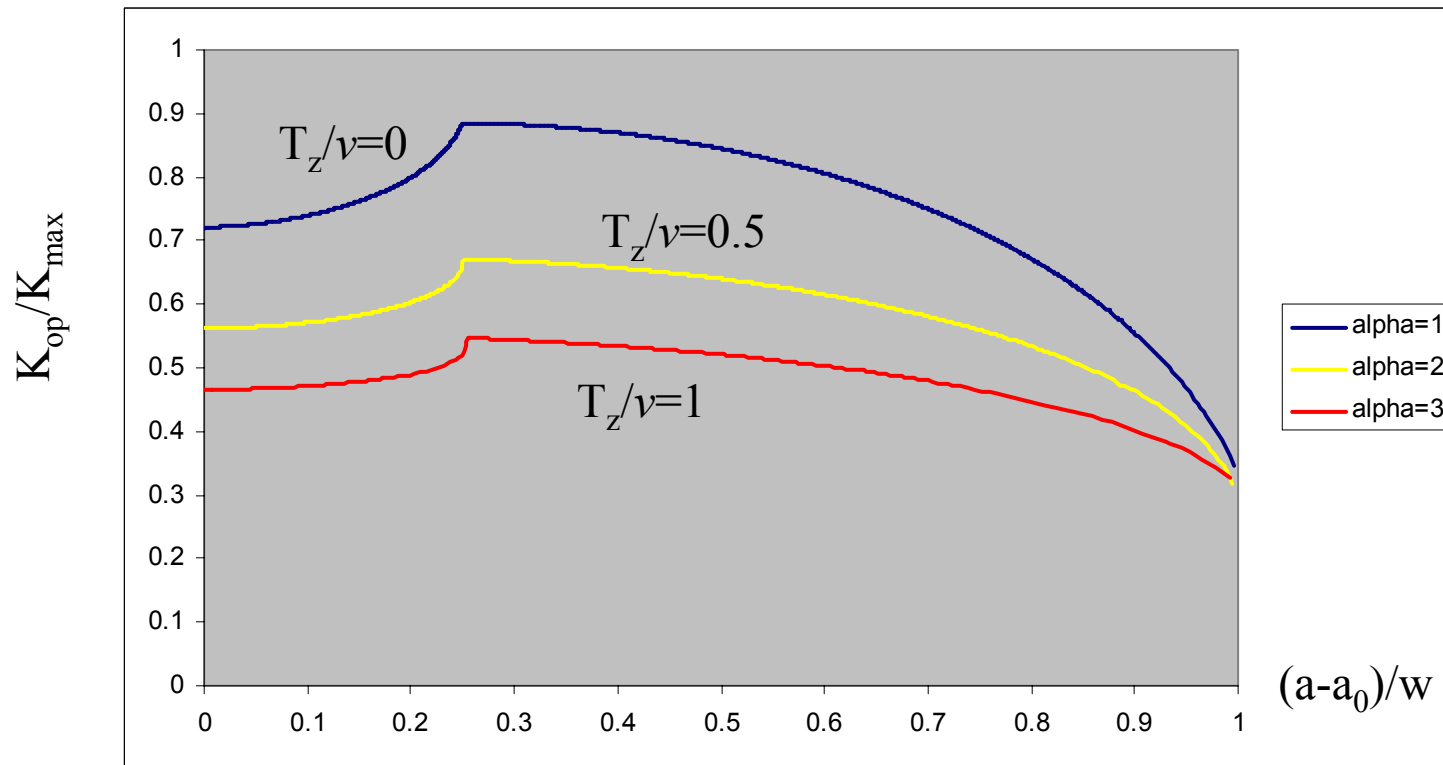
566 cycles

159 cycles

The crack propagation rate following an overload, under different 3D constraints

Plastic Zone model

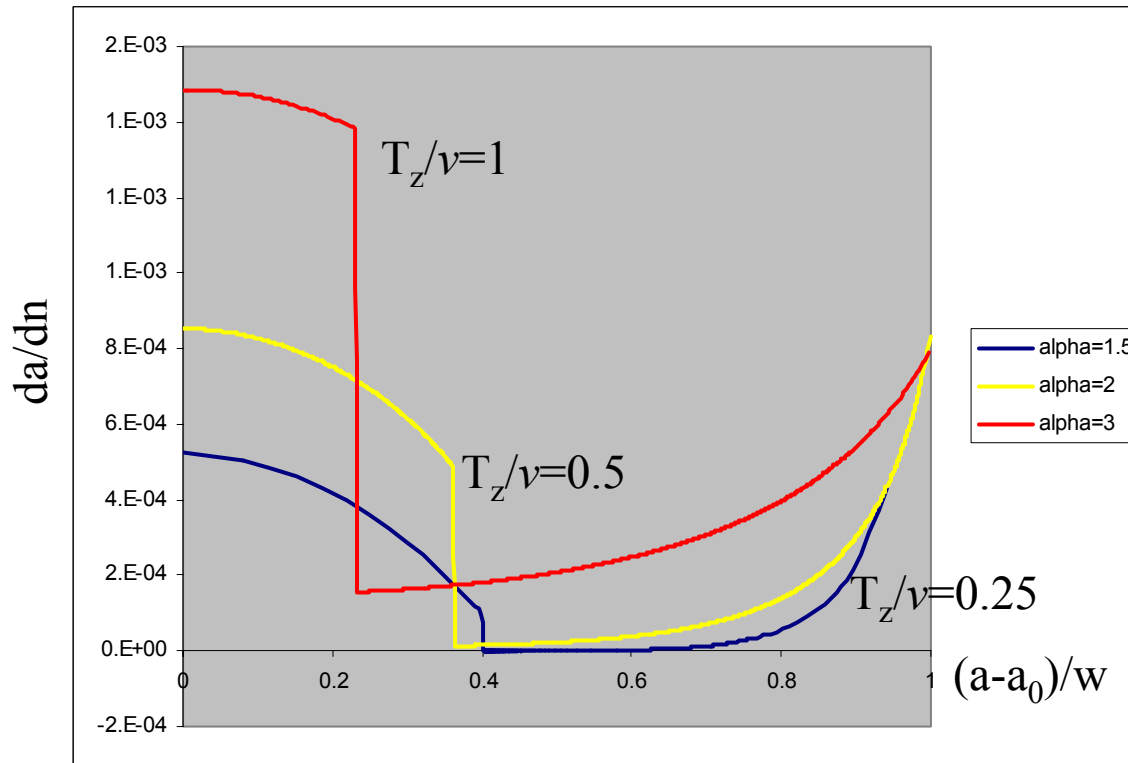
2024-T3 AL



The crack opening SIF following an overload, under different 3D constraints

Plastic Zone model

2024-T3 AL



$$K_{1\max}=869.63 \text{ MPa mm}^{1/2}$$

$$R_m=1.8$$

$$R_1=-0.5$$

$$R=0.396$$

$$C=2.383E-11$$

$$n=3.2$$

1,923,201 cycles

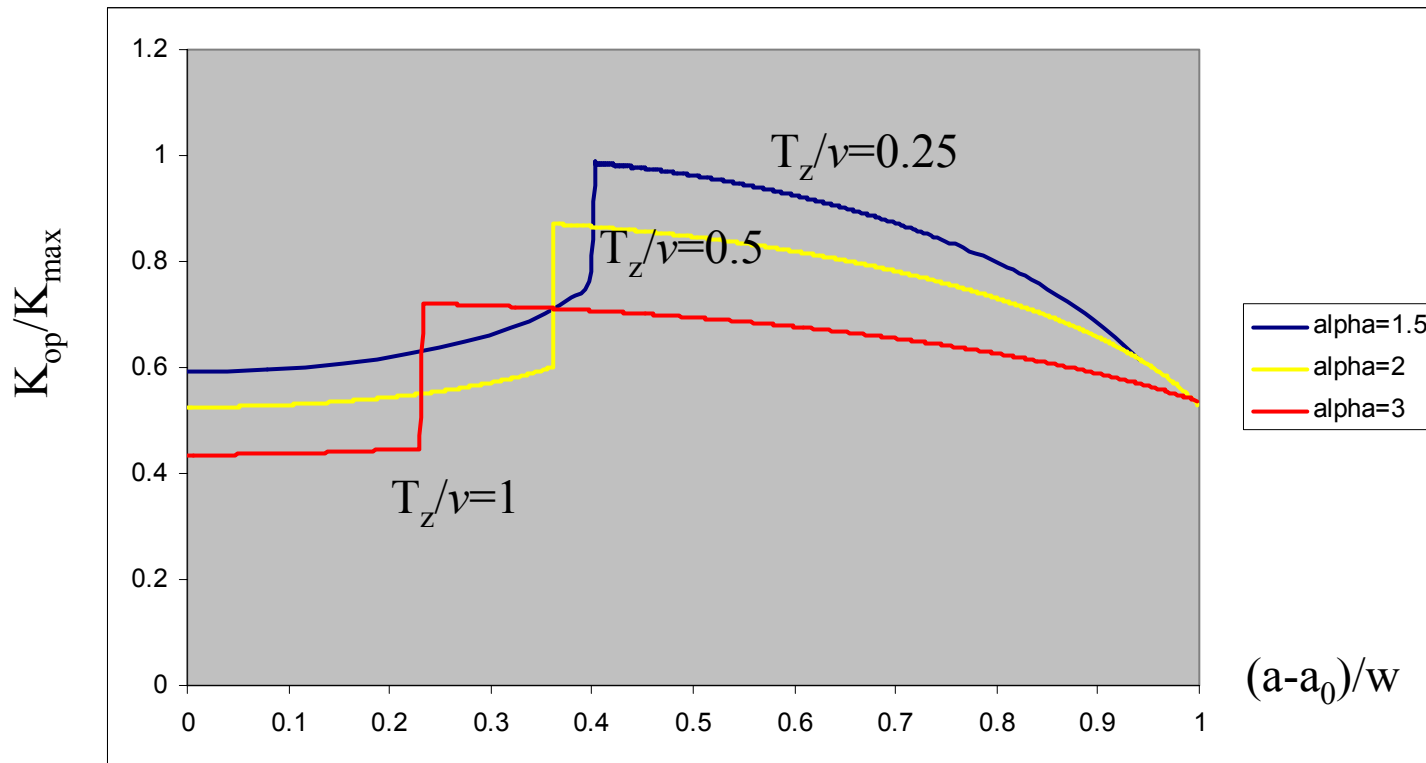
7,489 cycles

879 cycles

The crack propagation rate following an overload under different 3D constraints

Plastic Zone model

2024-T3 AL



The crack opening SIF following an overload, under different 3D constraints

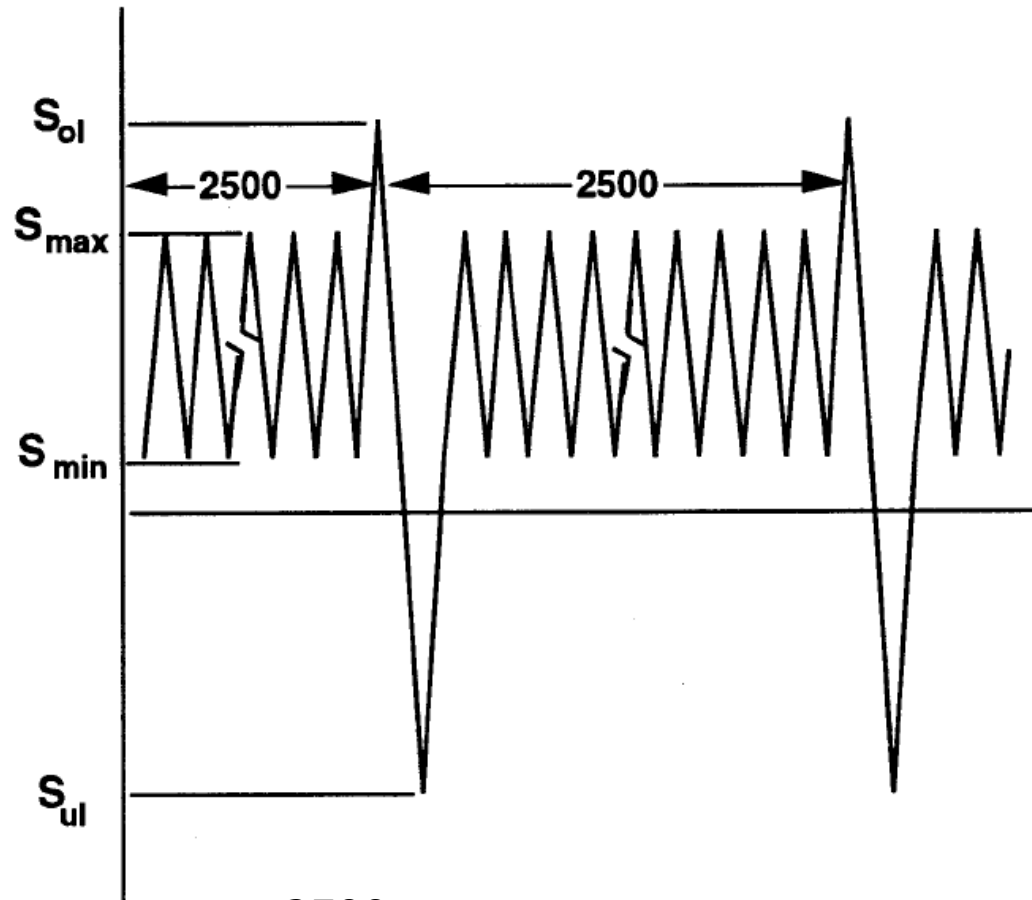
Plate with a center crack

2024-T3 AL

Maximum stress (S_{\max})	68.94 MPa
Minimum stress (S_{\min})	1.38 MPa
Stress ratio	0.02
Stress ratio of the overload	0
C	2.382 e-11
n	3.2
Yield Strength	365.42 MPa
Overload ratio (R_m)	1.5, 1.75
Initial crack length($2a_o$)	25.4 mm

Plate with a center crack

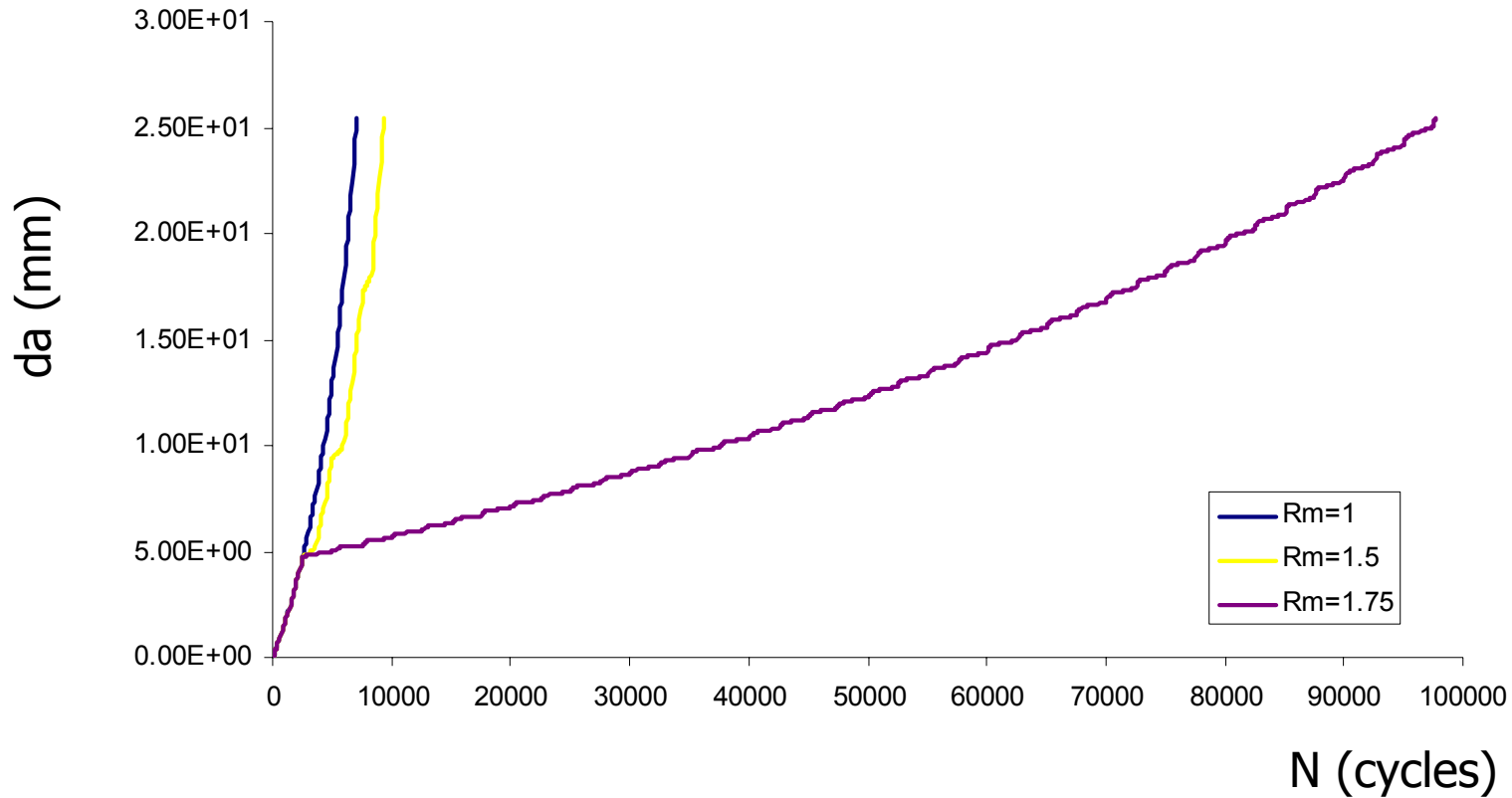
Overload spectra



the overload repeats at every 2500
constant amplitude load cycles

The stress ratio of the overload
 $R_o = S_{ul} / S_{ol}$

Plate with a center crack



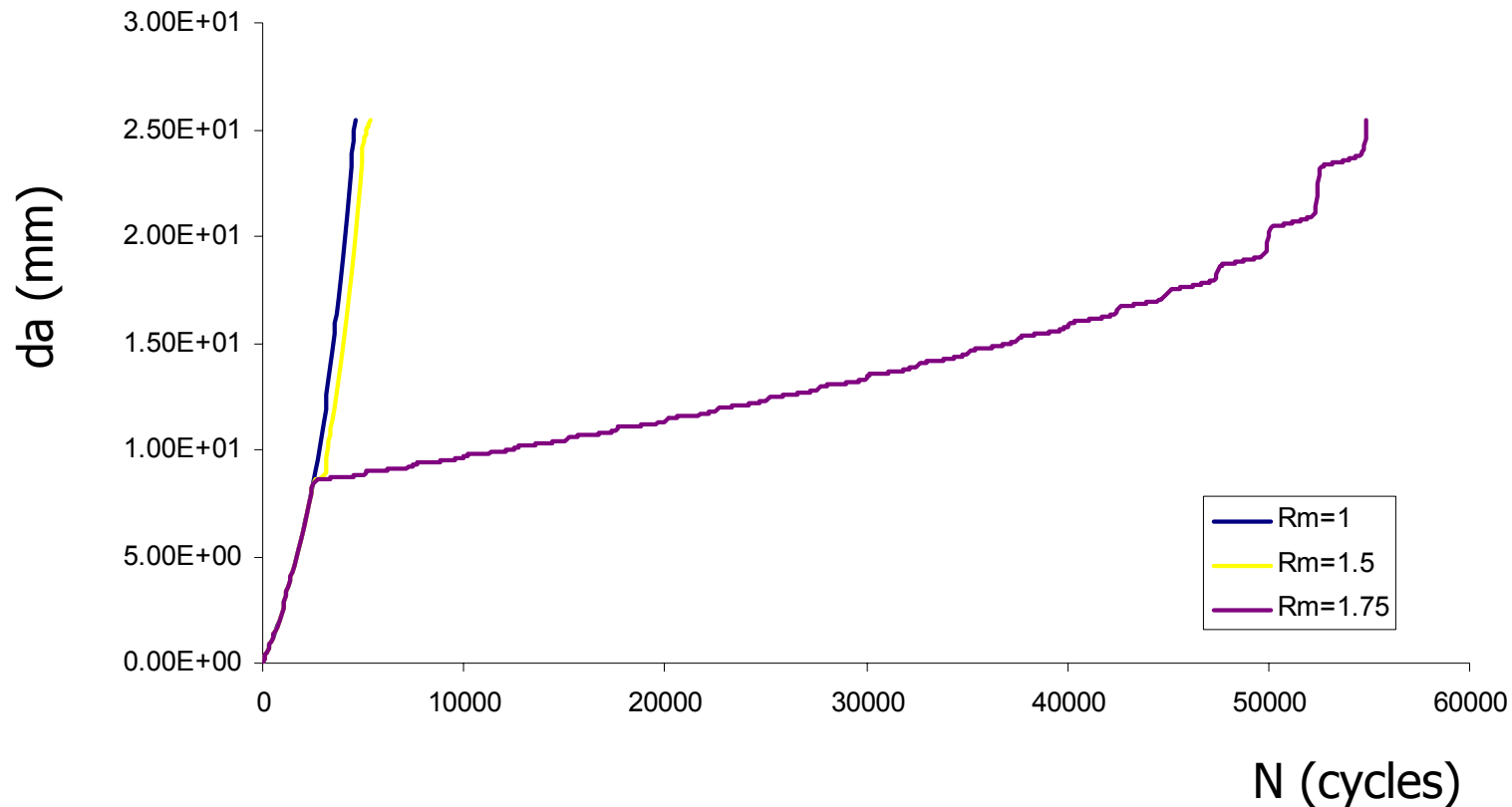
Plane stress, $T_z/\nu=0$

7,068 cycles

9,351 cycles

97,761 cycles

Plate with a center crack



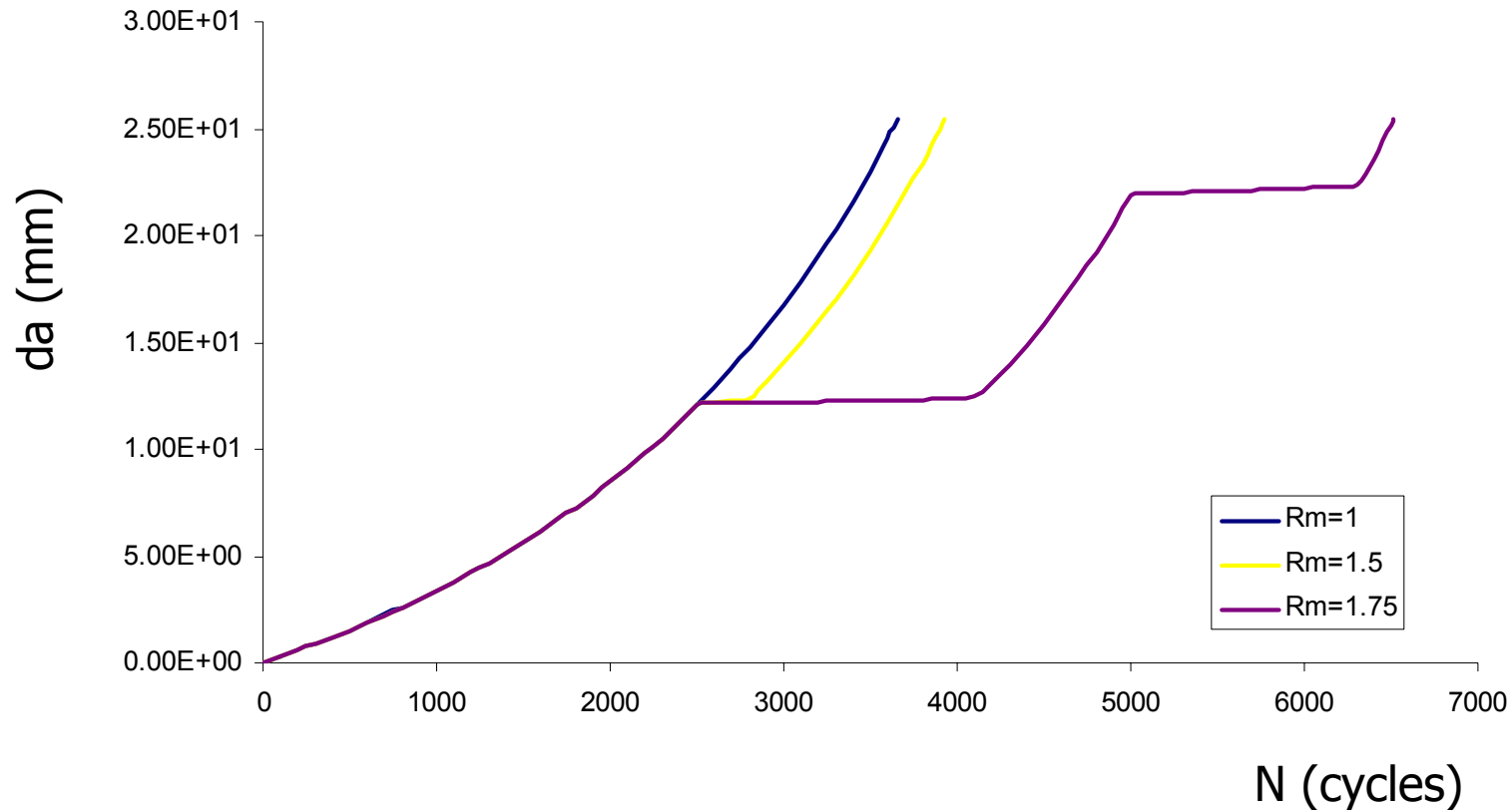
$T_z/v=0.5$

4,573 cycles

5,394 cycles

54,820 cycles

Plate with a center crack



Plane strain, $T_z/\nu=1$

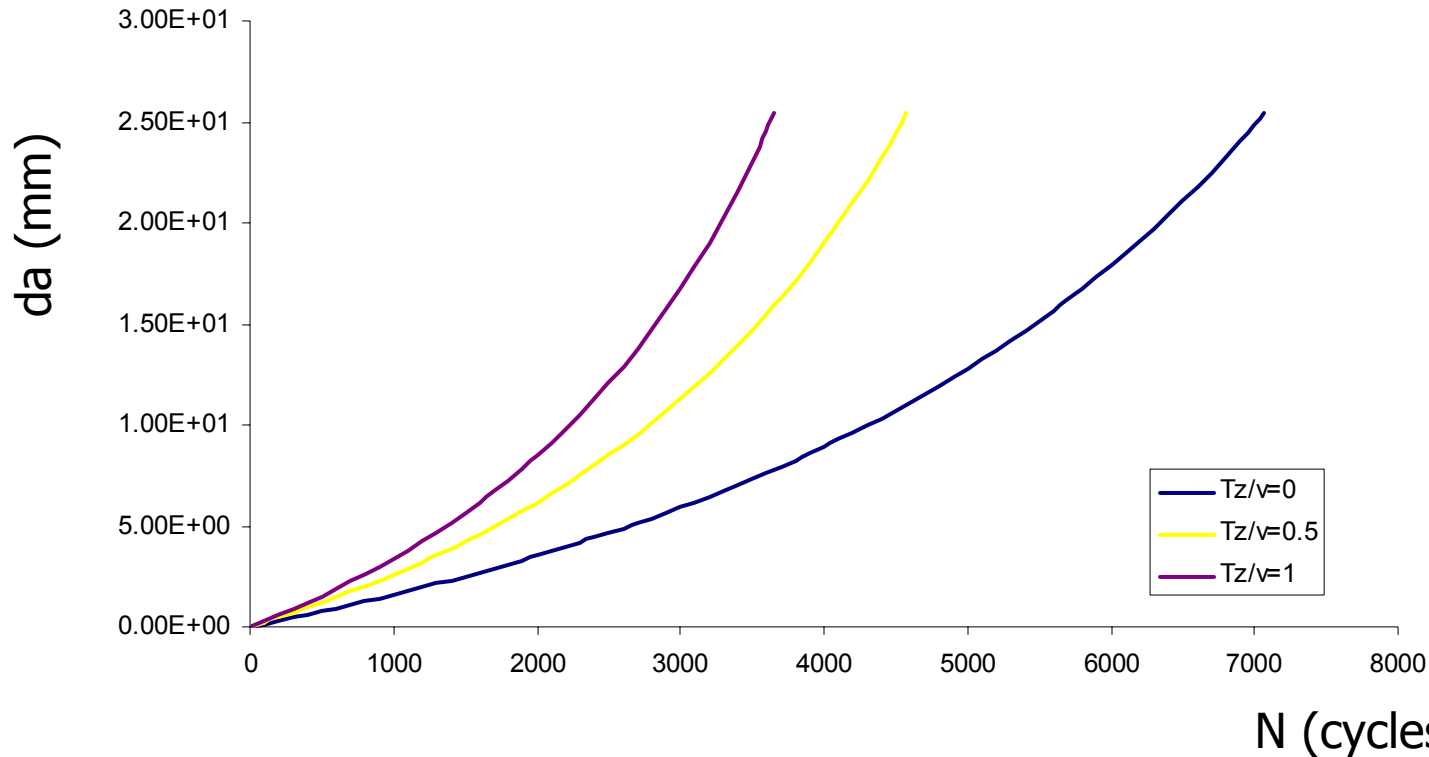
3,657 cycles

3,929 cycles

6,517 cycles

Plate with a center crack

The effect of the stress status



Constant amplitude load, $R_m=1$

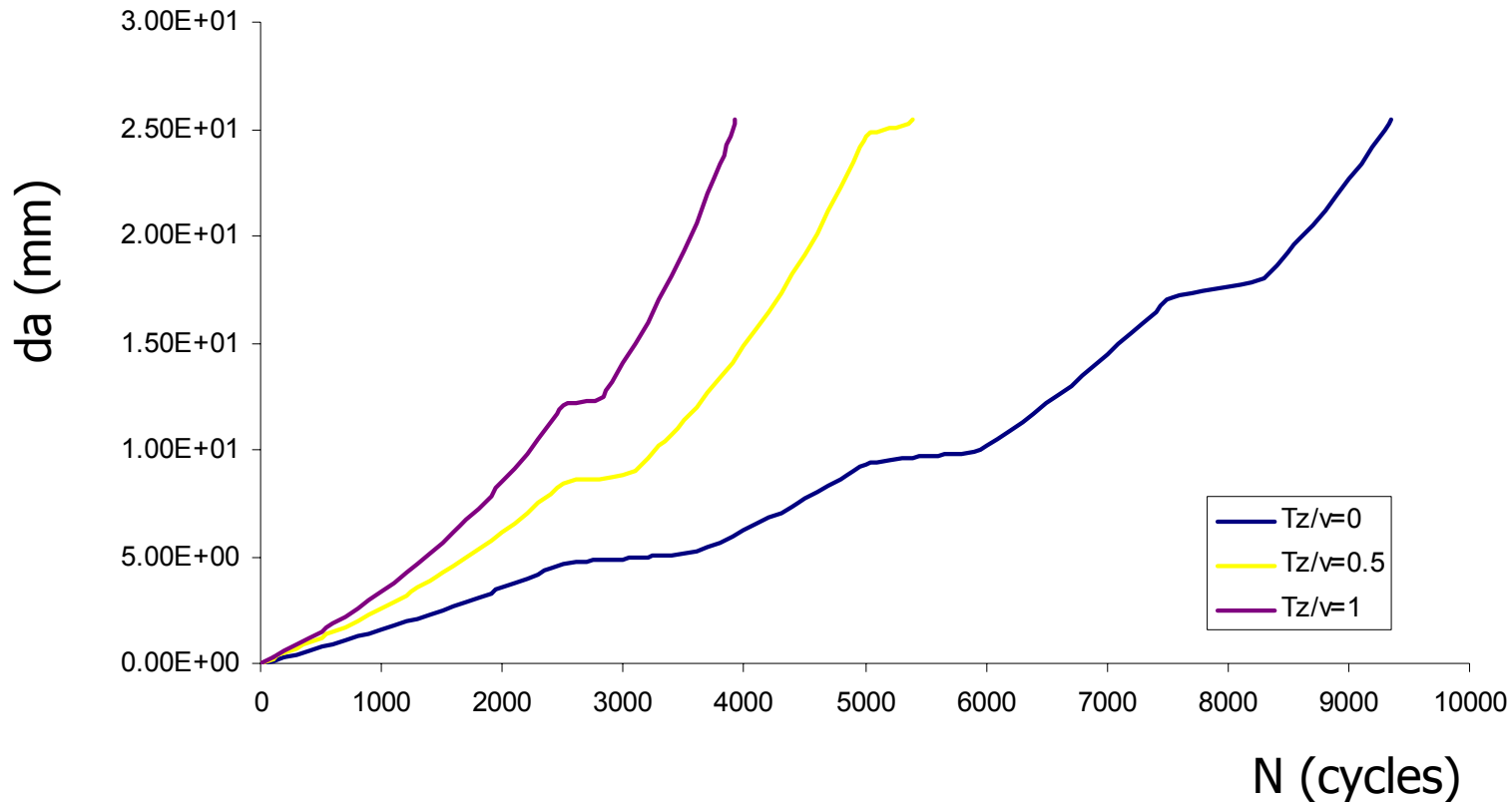
7,068 cycles

4,573 cycles

3,657 cycles

Plate with a center crack

The effect of the stress status



Overload spectrum, $R_m=1.5$

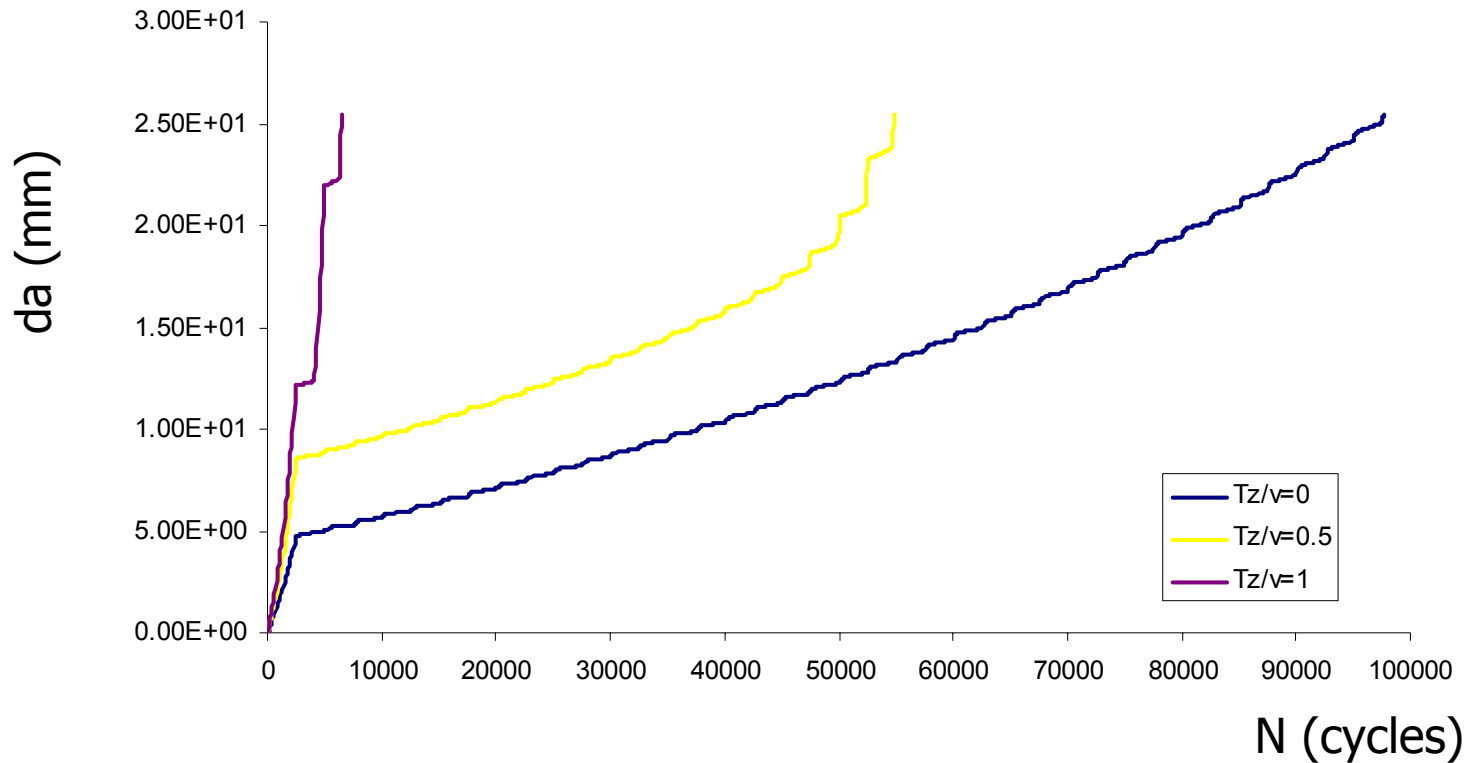
9,351 cycles

5,394 cycles

3,929 cycles

Plate with a center crack

The effect of the stress status



Overload spectrum, $R_m=1.75$

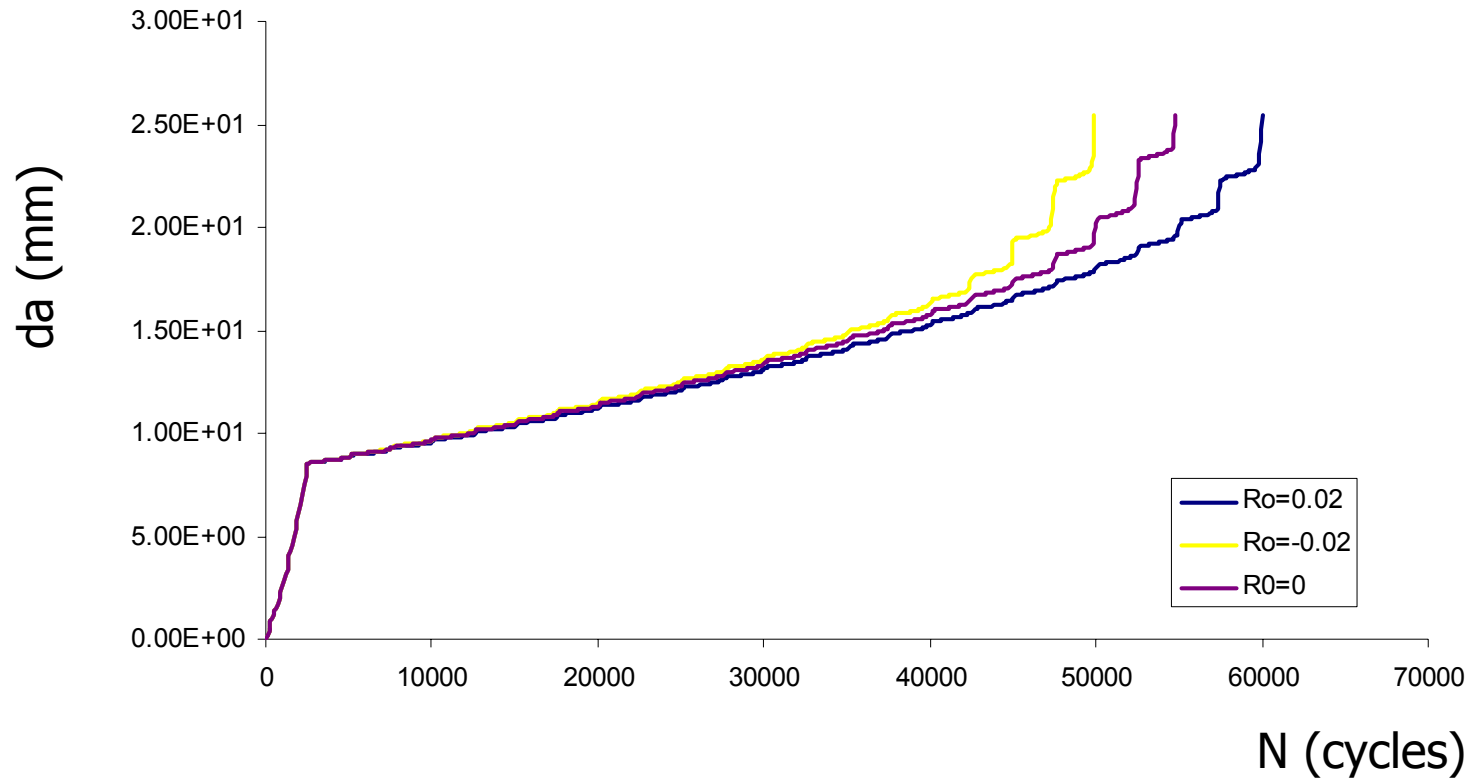
97,761 cycles

54,820 cycles

6,517 cycles

Plate with a center crack

The effect of the stress ratio of the overload



Overload spectrum, $T_z/v=0.5$, $R_m=1.75$

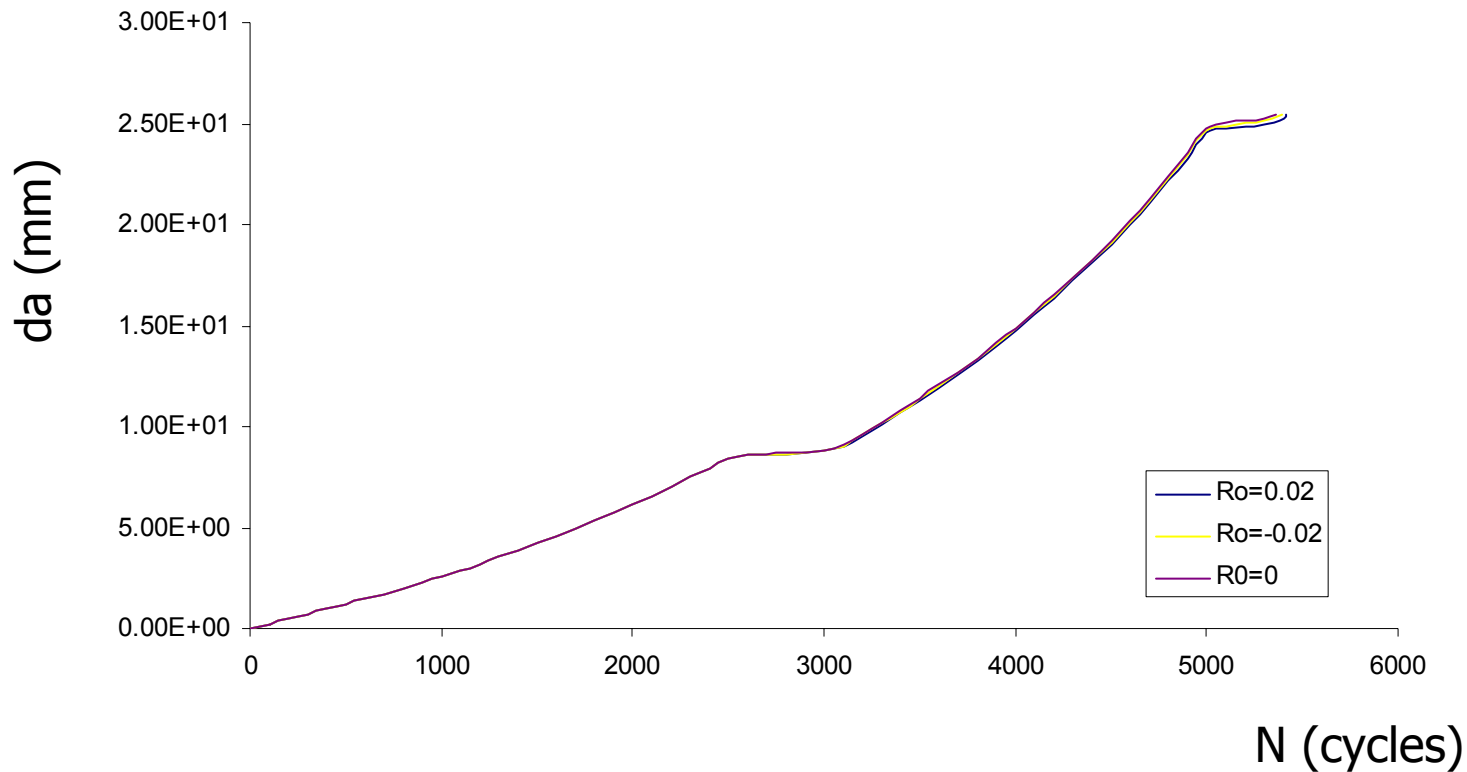
59,999 cycles

49,909 cycles

54,820 cycles

Plate with a center crack

The effect of the stress ratio of the overload



Overload spectrum, $T_z/v=0.5$, $R_m=1.5$

5,419 cycles

5,358 cycles

5,394 cycles

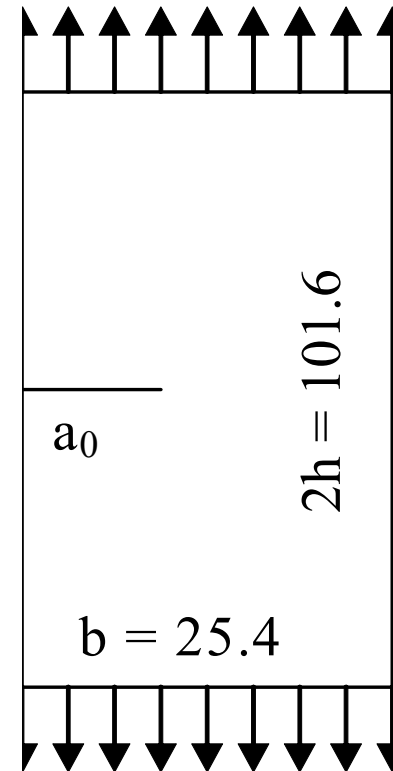
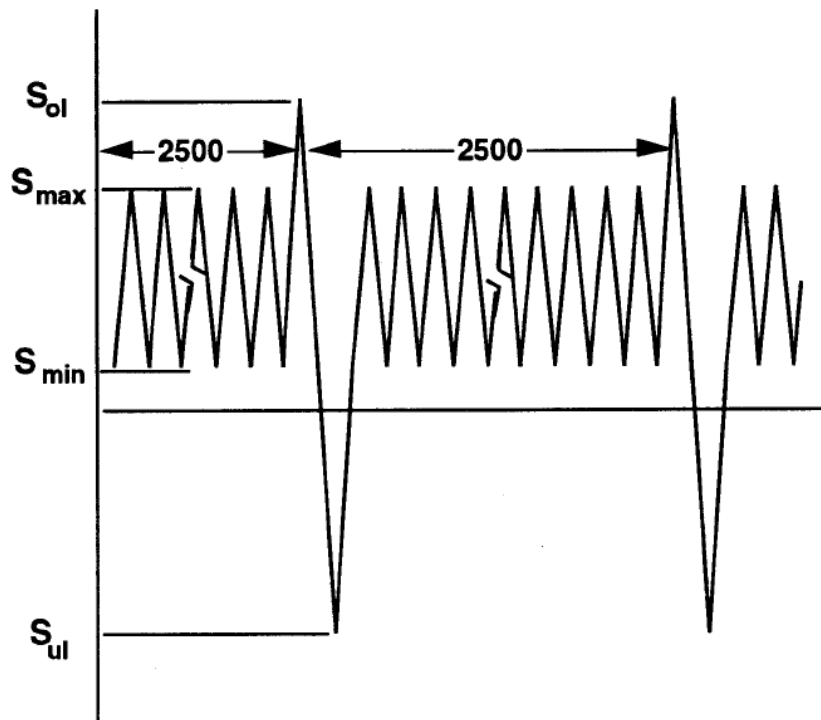
Plate with an edge crack

2024-T3 AL

Maximum stress (S_{\max})	68.94 MPa
Minimum stress (S_{\min})	1.38 MPa
Stress ratio	0.02
Stress ratio of the overload	-0.02
C	2.382 e-11
n	3.2
Yield Strength	365.42 MPa
Overload ratio (R_m)	1.25, 1.5
Initial crack length($2a_o$)	2 mm

Plate with an edge crack

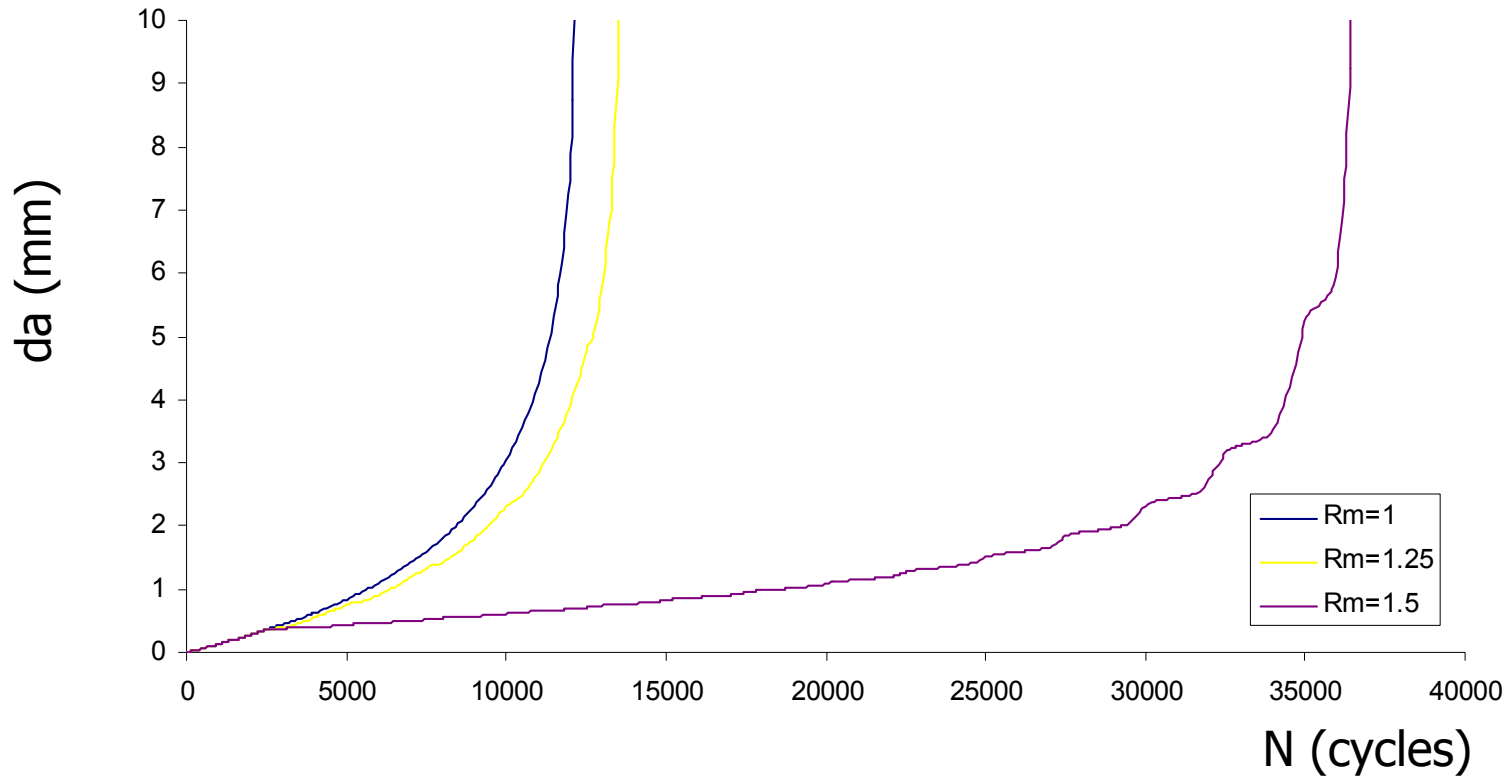
Overload spectra



the overload repeats at every 2500
constant amplitude load cycles

The stress ratio of the overload
 $R_o = S_{ul} / S_{ol}$

Plate with an edge crack



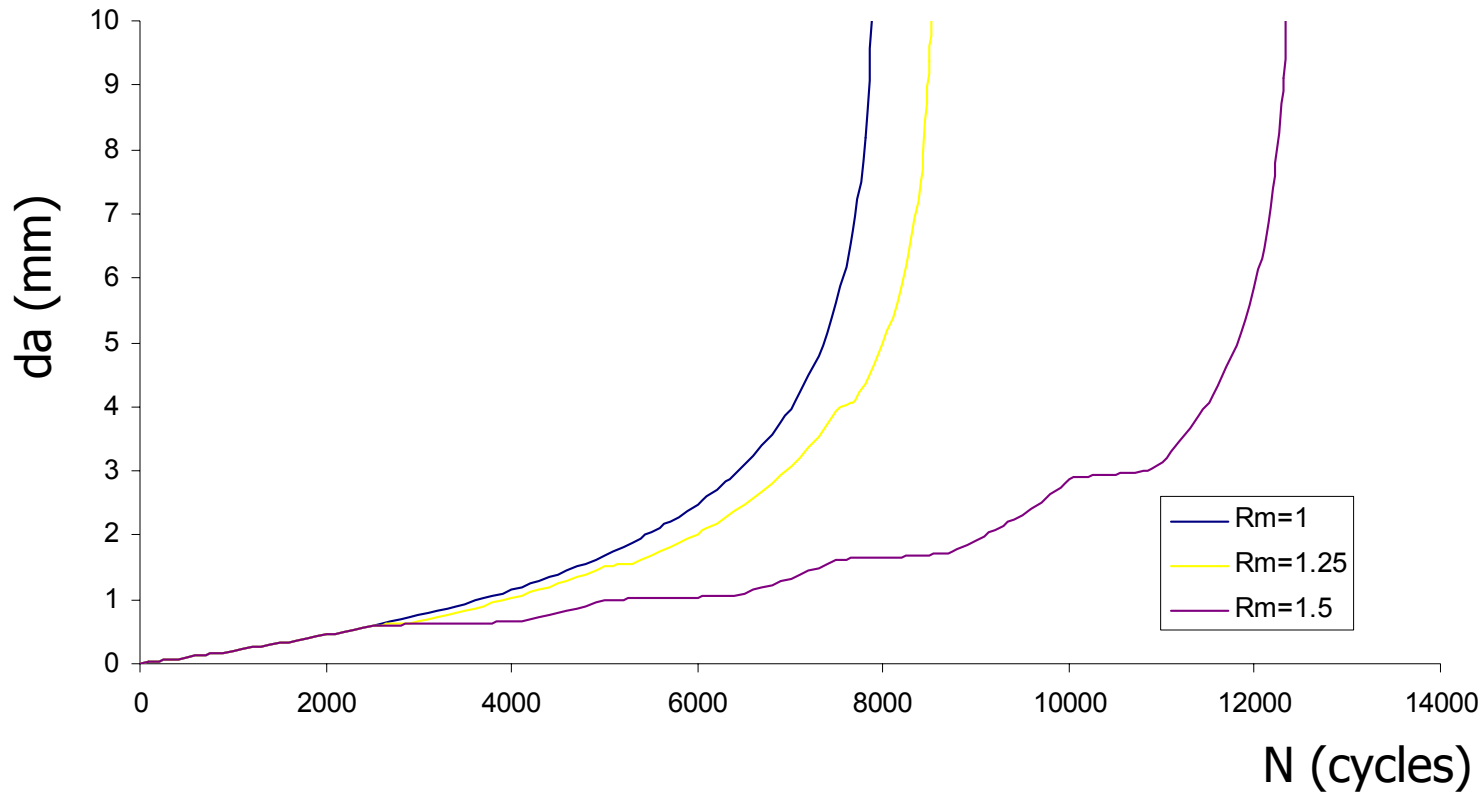
$T_z/v=0$

12,161 cycles

13,528 cycles

36,435 cycles

Plate with an edge crack



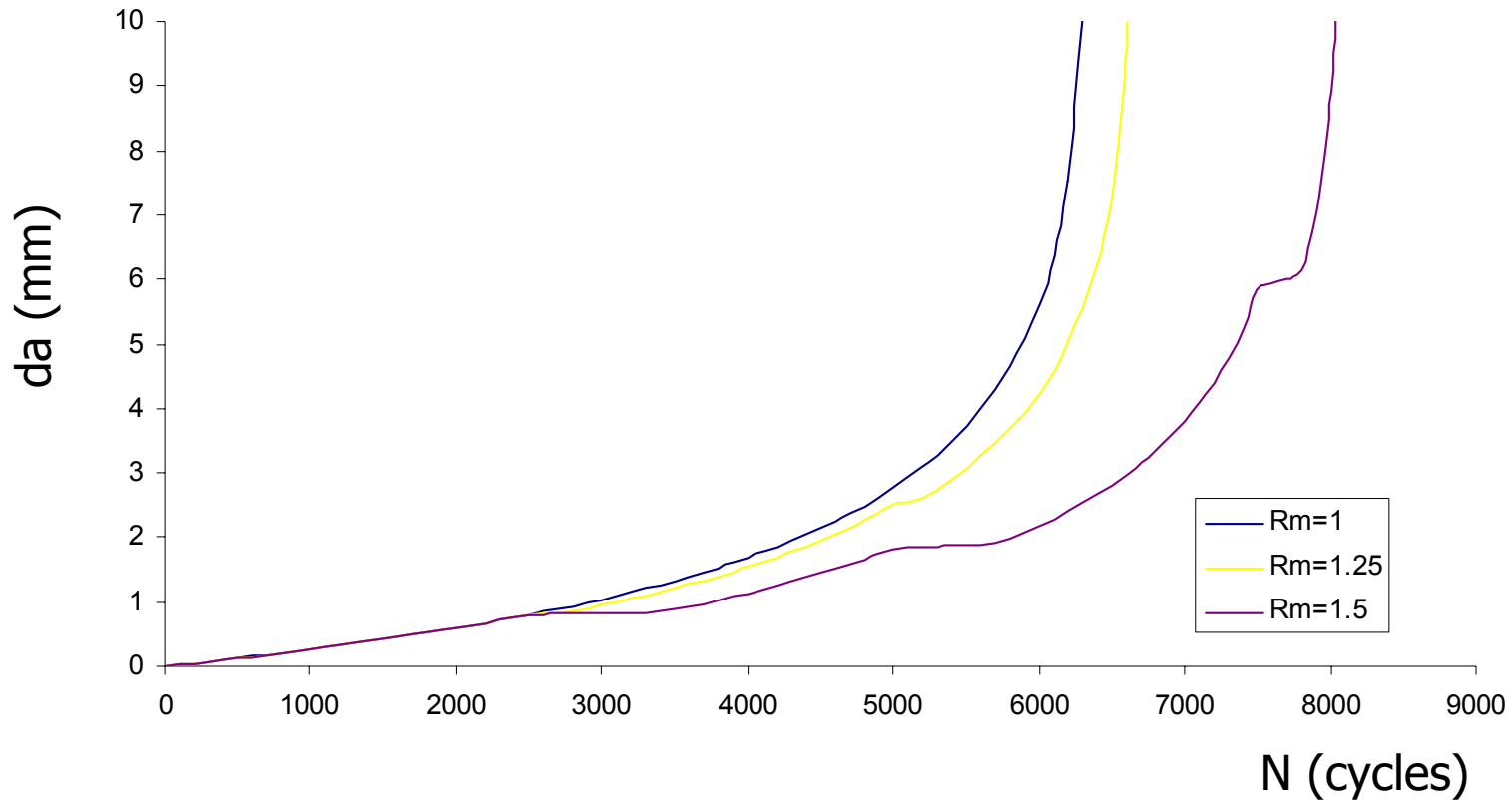
$T_z/v=0.5$

7,871 cycles

8,519 cycles

12,329 cycles

Plate with an edge crack



$T_z/v=1$

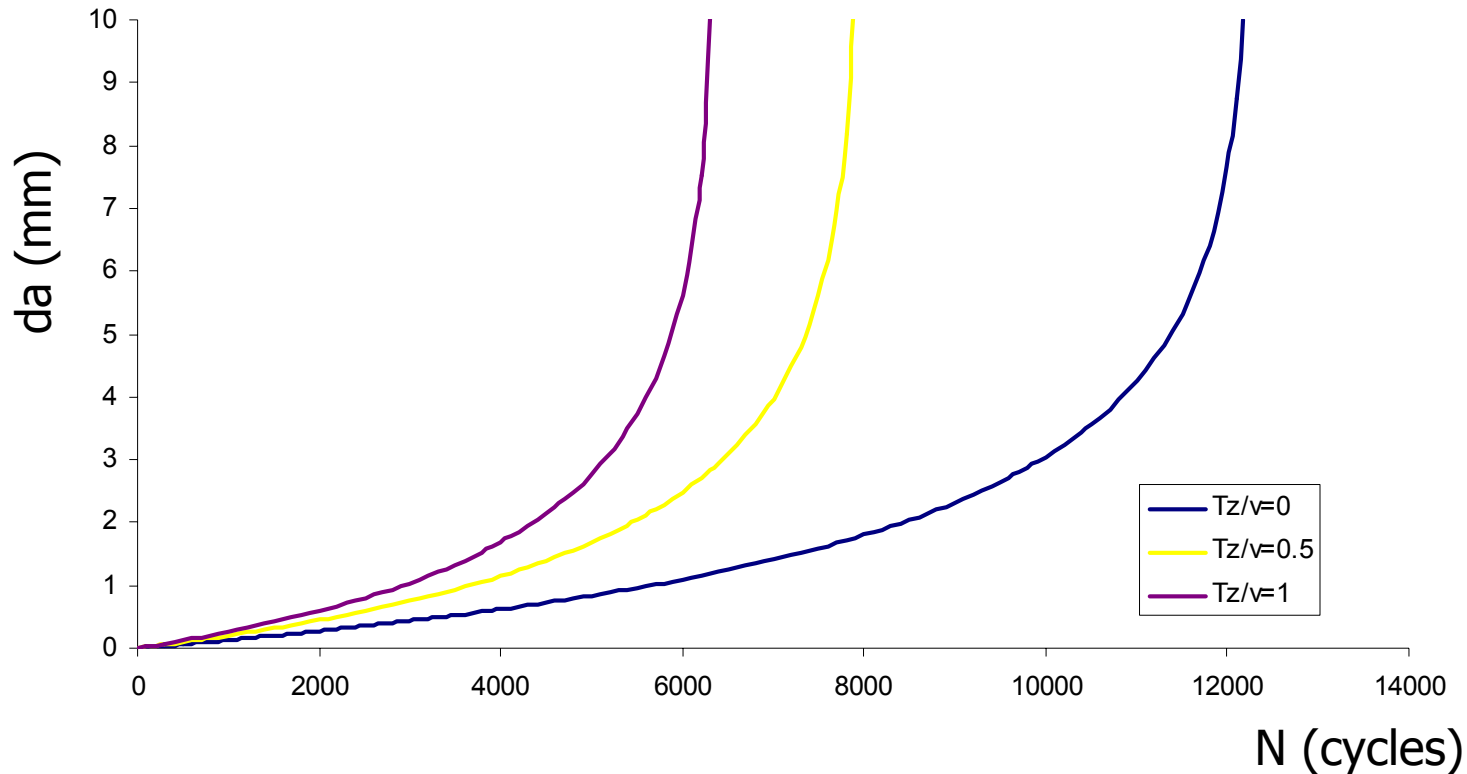
6,295 cycles

6,609 cycles

8,029 cycles

Plate with an edge crack

The effect of the stress status



Constant amplitude load, $R_m=1$

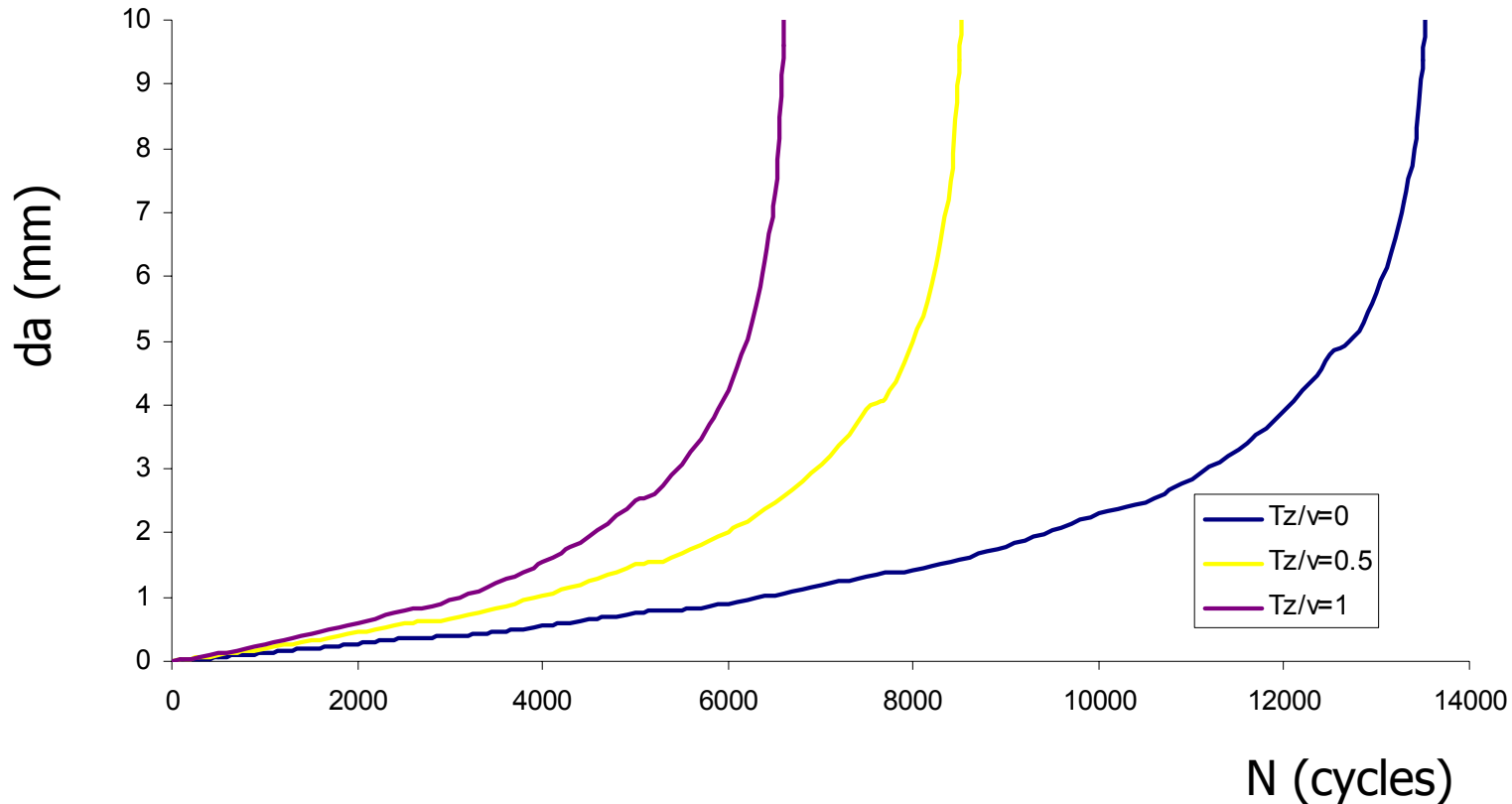
12,165 cycles

7,871 cycles

6,295 cycles

Plate with an edge crack

The effect of the stress status



Overload spectrum, $R_m=1.25$

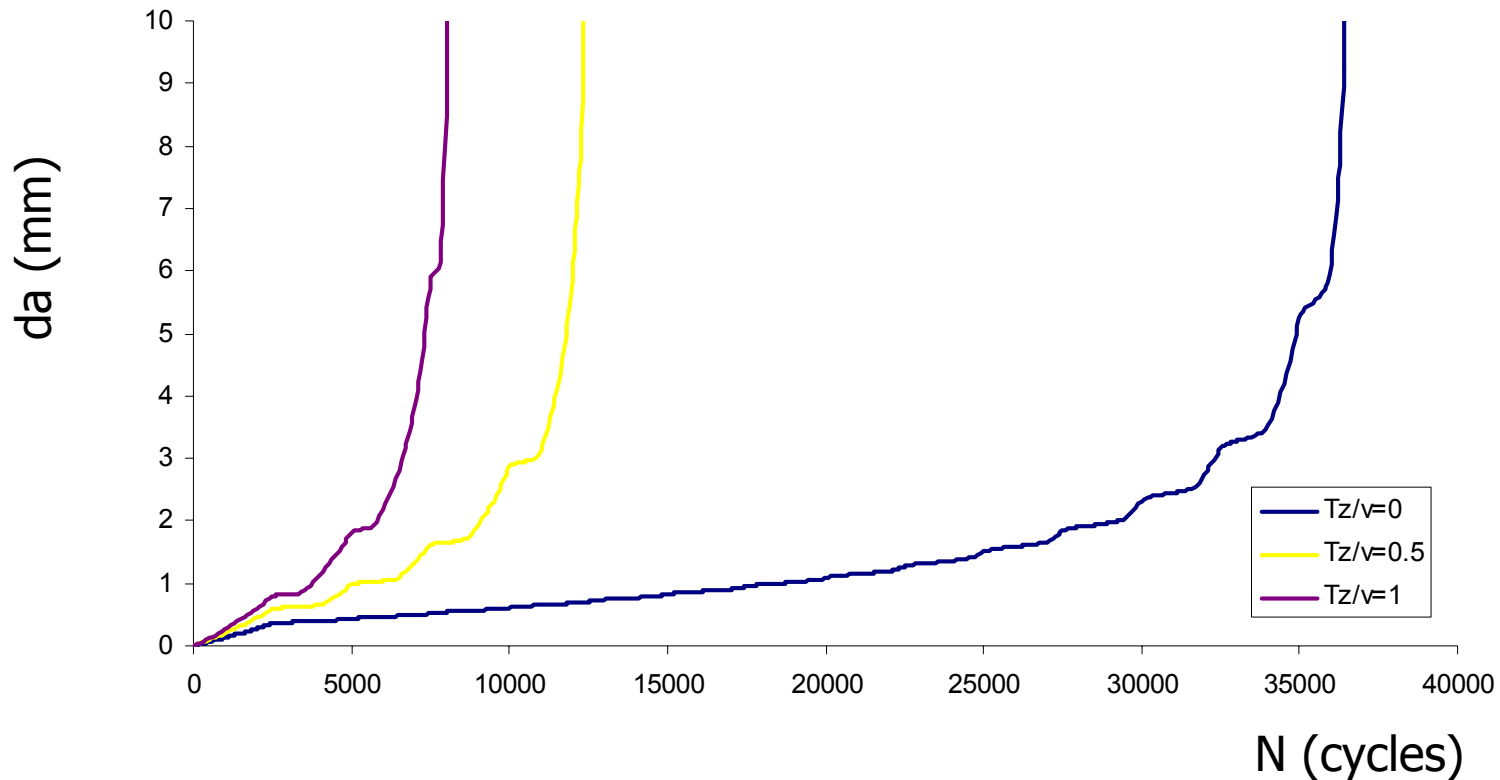
13,528 cycles

8,519 cycles

6,609 cycles

Plate with an edge crack

The effect of the stress status



Overload spectrum, $R_m=1.5$

36,435 cycles

12,329 cycles

8,029 cycles